Ultrasonics 54 (2014) 461–470

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Analysis of the modulated acoustic radiation-force profile for a dual-beam confocal geometry

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ARTICLE INFO

Article history:
Received 2 April 2013
Received in revised form 9 July 2013
Accepted 10 July 2013
Available online 19 July 2013

Keywords:
Acoustic radiation force
Quasilinear theory
Confocal beams

ABSTRACT

A localized modulated radiation force can be produced when two confocal ultrasound beams of nearly equal frequencies interfere in an attenuating medium such as tissue. It is well-established that this force generates both shear and longitudinal waves. By scanning the focal point over a plane and observing the propagation of these waves, the mechanical properties of the medium can be imaged. In this paper, the modulated radiation force is analytically derived in the case of attenuating media, by expanding on the theory of ultrasound-stimulated-vibro-acoustography (USVA) for lossless media. Furthermore, weak nonlinearities are considered in the formulation, since higher source pressures may prove to be necessary to improve the radiation-force profile – only the fundamental component is, however, studied in this paper. An analysis of the generated radiation force is performed and the effects of various parameters are investigated on its amplitude and spatial distribution. It will be shown that by carefully selecting the confocal geometry of the beams, as well as, the source pressure and center frequency, the spatial profile of the radiation force can be optimized. This, subsequently, could improve not only the resolution of the point-spread-function in USVA, but also, the profile of the shear waves in elastography applications.

1. Introduction

A wide class of elastography methods, aiming at measuring and visualizing the elastic properties of tissue [1–9] are based on the remote generation and propagation of longitudinal or shear waves. In a simple attenuating medium, two confocal quasi-CW ultrasound beams of slightly different frequencies can be used to generate, in the region of intersection (focal zone), a localized modulated radiation force at the difference frequency (typically in the low-kHz range). Such a scheme is illustrated in Fig. 1. Fatemi and Greenleaf [3,4] demonstrated that in response, the region emits low-frequency longitudinal waves that depend on the radiation force and the mechanical properties of the medium. In their method, known as *ultrasound-stimulated vibro-acoustography* (USVA), a high-resolution qualitative image of the elastic properties of tissue is produced by scanning the region of interest and assigning each time the amplitude of the emitted longitudinal wave (recorded externally by a hydrophone) to the corresponding pixel of the image output.

In a viscoelastic medium, the localized modulated radiation force, additionally to the longitudinal waves [3–6] can give rise to low-frequency shear waves [7–13] which are known to travel significantly slower in tissue than longitudinal waves [14]. By monitoring the propagation characteristics of these waves, quantitative information of the viscoelastic properties of the medium could be provided. This has been the topic of extensive research in the field of shear wave-based elastography. The shear displacement of the excited medium can be shown to depend on the viscoelastic Green's function and the modulated radiation force, by means of a spatiotemporal convolution [15].

The purpose of this paper is to analytically derive the modulated acoustic radiation force produced by two ultrasound confocal sources when viscous losses are considered (attenuating medium) and study its characteristics, as well as, its influence on the emitted longitudinal and shear waves that will be generated in response. The adopted theoretical formulation expands on the USVA theory for two confocal beams in the case of lossless media [4,5], by also including absorption in the analytical model. This, consequently, implies the generation and propagation of shear waves, in addition to the propagated longitudinal waves studied in the above works. Therefore, in the undertaken analysis, both phenomena will be considered and ways to optimize the radiation-force profile will be examined in order to improve both the shear and longitudinal-wave profiles.

As will be also shown, one such way may be that of increasing the ultrasound source pressure. Furthermore, in soft tissue, where the frequency-dependent attenuation coefficient can be quite high

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http://dx.doi.org/10.1016/j.ultras.2013.07.009

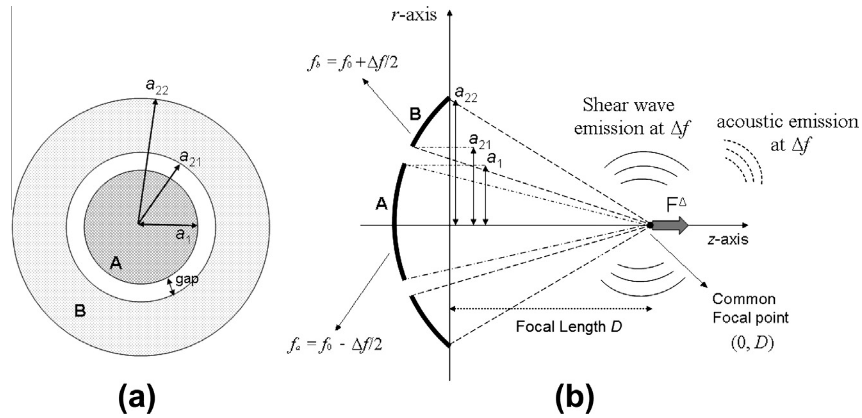

Fig. 1. Showing (a) a cross-section on the focal plane and (b) an axial view of the source system consisting of two coaxial confocal ultrasound transducers A and B. Transducer A (inner) is a confocal circular disk of aperture radius a_1 and transducer B (outer) is a confocal annular disk with inner and outer radii a_{21} and a_{22}, respectively. The dynamic radiation force produced by the interference of the two beams is also shown at the common focal point.

[14], relatively large source pressures may be needed to create sufficient radiation force for shear waves to be detected several wavelengths away from the source and for the local signal-to-noise ratio (SNR) to remain sufficiently high under conditions of noise [13]. These higher source pressures can, subsequently, give rise to nonlinear phenomena [13,14,16]. For the above reasons, weak nonlinearities are assumed in the theoretical model and the quasilinear theory [16] will be employed, such that the components of the acoustic pressure at the fundamental and second-harmonic frequencies will be taken into account. However, only the fundamental component will be studied in this manuscript – the implications of the second-harmonic radiation force will be examined in a subsequent paper.

Significant research had been conducted in the 1980s in the development of a theoretical formulation for focusing sources based on the parabolic approximation and quasilinear theory [17–22]. Tjotta and Tjotta [17] were the first ones, followed by Lucas and Muir [18,19] to derive analytical expressions for the fundamental and second-harmonic pressure fields in a quasilinear lossless focusing system. These expressions were expanded by Aanonsen et al. [22] for dissipative fluids and strong nonlinear conditions (shock formation). In this paper, the modulated radiation force will be analytically derived in the case of an attenuating medium by further expanding on these works.

The properties of the derived radiation force will be studied and the effects of several source parameters on its amplitude and spatial distribution will be examined. Specifically, it will be investigated if and how, the confocal geometry of the two beams and certain source parameters can be appropriately selected such that the generated radiation force is improved, with the aim of improving in turn, the estimation of the viscoelastic properties of tissue. It will be demonstrated that by carefully adjusting the aperture radii, focal length, source pressure and center frequency of the dual-beam system, the profile of the force field could be optimized, not only with respect to its amplitude, but also with respect to its spatial localization around the focal point. It will be also shown that by appropriately accounting for these parameters, the spatial characteristics of the longitudinal and shear waves induced for elasticity imaging applications could be improved. For this reason, the point-spread-function of an USVA imaging system will be studied, as well as, the spatiotemporal profile of the generated shear displacement in radiation force-based elastography methods. The associated tradeoffs in the selection of the optimal geometry and source parameters will be also discussed.

2. Theoretical formulation

The radiation-force imaging system is considered to be composed of two spherically confocal coaxial beam sources A and B, of common focal length, which interfere in the focal zone, as shown in Fig. 1. Beam-A (inner) is assumed to be a circular disk with an outer aperture radius a_1, while beam-B (outer) is defined as an annular disk with inner and outer aperture radii a_{21}, a_{22}, correspondingly. The two focused ultrasound beams A and B are excited at the same source pressure and at the angular frequencies $\omega_a = \omega_0 - \Delta\omega/2$ and $\omega_b = \omega_0 + \Delta\omega/2$, where $\omega_0 = 2\pi f_0$ and $\Delta\omega$ are the center and modulation angular frequencies, respectively, such that $\Delta\omega \ll \omega_0$ (f_0 being the center frequency) [3,4,11].

In the subsequent analysis, the pressure profiles of the two beams are derived based on the KZK parabolic-wave equation, which accounts for nonlinearity, absorption and diffraction [16,19–21]. The parabolic and quasilinear approximations are employed. For our dual-beam source, the parabolic-approximation condition implies that $k_a a_1 \gg 1$ and $k_b(a_{22} - a_{21}) \gg 1$ [17,18] where $k_i = \omega_i/c_0$ ($i = a, b$) is the wavenumber for each beam, respectively. The quasilinear condition considers that nonlinear components are only generated by the interaction of lower-order harmonics, i.e. the fundamental and the second-harmonic beams, since they are much stronger than the higher-order harmonics [19,22]. As discussed earlier, only the fundamental component will be presented in this paper, while the effects of the second harmonic on the generated radiation-force field will be studied in a subsequent paper.

For beam-A, the linearized solution of the parabolic KZK equation for an axisymmetric source that oscillates sinusoidally in time, can be described in cylindrical coordinates by [18,22]:

$$\underline{p}_a(r,z) = -j\frac{p_0\omega_a}{c_0 z} e^{jk_a z} e^{-\alpha z} e^{\frac{jk_a r^2}{2z}} \times \int_0^{a_1} \exp\left[\frac{jk_a x^2}{2}\left(\frac{1}{z}-\frac{1}{D}\right)\right] J_0\left(\frac{k_a r x}{z}\right) x dx \quad (1)$$

where $\underline{p}_a = \underline{p}_a(r, z{:}\omega)$ is the pressure phasor at frequency ω_a and $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind. The parameters c_0, p_0 and D denote the small-signal sound speed, the amplitude of the source pressure and the focal length, respectively. The latter defines the distance between the transducer and the location in the focal zone (see Fig. 1) where the linear acoustic pressure is maximized [14]. In (1), α describes the attenuation coefficient, which was assumed to have a square-law dependence with