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Reconstructing 3-D maps of the local viscoelastic properties using a finite-amplitude modulated radiation force

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ABSTRACT

A modulated acoustic radiation force, produced by two confocal tone-burst ultrasound beams of slightly different frequencies (i.e. 2.0 MHz ± $\Delta f/2$, where Δf is the difference frequency), can be used to remotely generate modulated low-frequency ($\Delta f \leqslant 500$ Hz) shear waves in attenuating media. By appropriately selecting the duration of the two beams, the energy of the generated shear waves can be concentrated around the difference frequency (i.e., $\Delta f \pm \Delta f/2$). In this manner, neither their amplitude nor their phase information is distorted by frequency-dependent effects, thereby, enabling a more accurate reconstruction of the viscoelastic properties. Assuming a Voigt viscoelastic model, this paper describes the use of a finite-element-method model to simulate three-dimensional (3-D) shear-wave propagation in viscoelastic media containing a spherical inclusion. Nonlinear propagation is assumed for the two ultrasound beams, so that higher harmonics are developed in the force and shear spectrum. Finally, an inverse reconstruction algorithm is used to extract 3-D maps of the local shear modulus and viscosity from the simulated shear-displacement fields based on the fundamental and second-harmonic component. The quality of the reconstructed maps is evaluated using the contrast between the inclusion and the background and the contrast-to-noise ratio (*CNR*). It is shown that the shear modulus can be accurately reconstructed based on the fundamental component, such that the observed contrast deviates from the true contrast by a root-mean-square-error (*RMSE*) of only 0.38 and the *CNR* is greater than 30 dB. If the second-harmonic component is used, the *RMSE* becomes 1.54 and the corresponding *CNR* decreases by approximately 10–15 dB. The reconstructed shear viscosity maps based on the second harmonic are shown to be of higher quality than those based on the fundamental. The effects of noise are also investigated and a fusion operation between the two spectral components is applied to enhance the reconstruction quality. Finally, a modified shear-wave spectroscopy technique, shown to be more robust to noise, is described for the estimation of the viscoelastic properties inside and outside the spherical inclusion under conditions of increased noise.

1. Introduction

In a simple viscoelastic medium, two confocal CW ultrasound beams of slightly different frequencies (i.e. 2.0 MHz ± $\Delta f/2$, where Δf is the difference frequency) can be used to remotely generate a localized modulated radiation force at the difference frequency (typically in the low-kHz range, i.e. $\Delta f \leqslant 500$ Hz). Fatemi and Greenleaf [1] demonstrated that in response to the generated radiation force, the excited region (focal zone) of a lossless (ideal) medium emits low-frequency ($\Delta f \leqslant 500$ Hz) longitudinal waves that depend on the radiation force and the mechanical properties of the medium. If, in addition, viscous losses are also taken into account, then the radiation force can also give rise to the generation of low-frequency shear waves, which are known to travel significantly slower in tissue than longitudinal waves [2]. By monitoring the propagation characteristics of the shear waves, quantitative information of the viscoelastic properties of the medium could be provided. This has been the topic of extensive research in the field of shear wave-based elastography [3–9]. Furthermore, by appropriately selecting the emission duration of the radiation force (in our model it has been selected to be equal to 1.5 cycles at the difference frequency Δf, as it will be seen in Section 4), the generated shear waves can be shown to exhibit a relatively narrowband frequency spectrum, such that the shear-wave energy is concentrated within the frequencies $\Delta f \pm f_n$, where $f_n < \Delta f/2$. Consequently, the amplitude and the phase information of the shear-wave spectrum are not distorted by any frequency-dependent effects, thereby, providing more reliable estimates of the viscoelastic properties of the medium [5].

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Finite element methods are efficient numerical techniques that can find approximate solutions to partial differential equations and their systems over complicated domains. However, they have not been used extensively in shear elastography. In the work by Chen et al. [10], a 2-D FEM model was proposed to simulate the shear-wave propagation in homogeneous elastic media for magnetic resonance elastography (MRE) applications. Palmeri et al. [11,12] developed a more complete 3-D FEM model to study the dynamic shear-displacement field in response to an impulsive acoustic radiation force (ARFI) excitation from a linear array transducer. Homogeneous tissues of varying stiffnesses (Young's moduli) were considered with and without stiffer spherical inclusions and the dependence of the shear response was demonstrated on the ultrasonic attenuation, density, Poisson's ratio and elasticity. However, the above method ignored the effects of viscosity, which has been shown to play an important role in such applications [5,6,12].

Giannoula and Cobbold [7] proposed a finite element method to simulate the propagation of narrowband shear waves in viscoelastic media, in response to the modulated radiation force resulting from two interfering focused beams of nearly equal frequencies, as described previously. Although the effects of shear viscosity were thoroughly studied in that model, two assumptions were adopted that may limit its clinical applicability: the assumption of a 2-D propagation model and that of a point-force excitation (spatial impulse). With respect to the first limitation, the associated Navier–Stokes equation is generally defined in the 3-D space, thereby suggesting the use of a 3-D FEM model to more accurately simulate the shear-wave propagation. Furthermore, the generated radiation force is not a spatial impulse, but it is rather characterized by a spatial 3-D distribution (body force). This means that additional force-field points in the 3-D neighborhood surrounding the focal point will also affect the propagation of such shear waves.

In this paper, the previously developed 2-D FEM model [7] will be expanded and 3-D FEM simulations will be conducted in order to more accurately simulate the propagation of shear waves that are generated in response to a volumetric (3-D) finite-amplitude modulated radiation force in viscoelastic media containing inclusions. It should be noted, that for reasons of computational saving and simplicity, spherical inclusions were considered in this paper. In the proposed FEM model, the associated partial differential equations (Navier–Stokes equation) will be numerically solved in the 3-D space using the Voigt model of viscoelasticity [3]. Furthermore, since in soft tissue the frequency-dependent attenuation coefficient can be quite high (>0.5 dB/cm MHz, as reported in Table 1.8 of [2]), relatively large ultrasound source pressures may be needed, such that the generated shear waves can be detected several wavelengths away from the source. Such source pressures (selected to be >370 kPa in our simulation model, as will be seen in Section 4) will cause nonlinear ultrasound propagation and will result in higher harmonics in the beam and force spectrum. An inverse reconstruction algorithm will be employed in order to obtain 3-D maps of the local shear modulus and viscosity based on both the fundamental and second-harmonic components of the radiation force. The performance of the reconstructed images will be assessed using the contrast and contrast-to-noise ratio between the background and inclusion. It should be noted, that the issue of reconstructing and evaluating 3-D images of both the local shear modulus and viscosity, based on the fundamental and second-harmonic component of the shear spectrum, to the best of our knowledge, has not been previously discussed in the literature. The effects of noise will be also studied and methods to improve the reconstruction quality will be discussed. One such method, more robust to noise, is an adaptation of the shear-wave spectroscopy technique by Deffieux et al. [13] in the 3-D space. The viscoelastic properties estimated using this method are shown to be more accurate even in the case of increased noise.

This paper is outlined as follows: in Section 2, the generation of the modulated shear waves in response to a finite-amplitude modulated radiation force is theoretically described. The proposed 3-D FEM model for simulating the propagation of these shear waves is presented in Section 3. The inverse algorithm for the reconstruction of 3-D maps of the local shear modulus and viscosity is described in the same section, as well as, the image quality metrics that will be used to evaluate the performance of the reconstruction algorithm. The results of our simulations with and without noise will be presented in Section 4, where a modified shear-wave spectroscopy technique will be also proposed for conditions of high noise. Finally, conclusions will be drawn in Section 5, where a discussion will be also provided on the findings of the proposed model.

2. Theory and background

As illustrated in Fig. 1, two intersecting tone-burst ultrasound beams A and B are assumed with slightly different frequencies, i.e. $f_a = f_0 - \Delta f/2$ and $f_b = f_0 + \Delta f/2$, where f_0 and Δf ($\Delta f \ll f_0$) denote the center and difference (modulation) frequencies, respectively. An infinite, isotropic, and homogeneous medium is considered, with tissue-like attenuating characteristics (its attenuation coefficient – at ultrasound frequencies – is given by $\alpha_n = \alpha_0 f^\gamma$, where α_0 is the attenuation coefficient and $\gamma \approx 1$). As noted earlier, nonlinear propagation is considered for the two ultrasound beams. Therefore, in the intersection zone, the acoustic radiation body-force contains low-frequency modulated components at the fundamental and harmonic modulation frequencies, i.e. at $n\Delta f$, for $n = 1$, 2, etc., which can be written as:

$$F_n^\Delta(\mathbf{r},t) = \frac{2\alpha_0 n^\gamma (f_a^\gamma + f_b^\gamma) I_n^\Delta(\mathbf{r},t)}{c} \tag{1}$$

as shown in [6]. In the above formula, $I_n^\Delta(\mathbf{r},t)$ is the dynamic component of the intensity vector at point $\mathbf{r}$ with coordinates (x,y,z) and c is the propagation speed of the incident longitudinal waves. The radiation force described in (1), practically, results from the transfer of momentum from the incident waves to the propagating medium due to the medium attenuation [5] and it can be further expanded as it is in [6]:

$$F_n^\Delta(\mathbf{r},t) = \frac{2\alpha_0 n^\gamma (f_a^\gamma + f_b^\gamma) p_{n,a}(\mathbf{r}) p_{n,b}(\mathbf{r}) H(t) H(D-t) \cos[n \cdot 2\pi\Delta f \cdot t + \Delta\phi_n(\mathbf{r})]}{\rho c^2} \tag{2}$$

where $p_{n,a}(\mathbf{r})$, $p_{n,b}(\mathbf{r})$ denote the nth harmonic pressure amplitudes of the individual ultrasonic beams at point $\mathbf{r}$ and ρ is the medium density. $H(.)$ denotes a Heaviside step function and $\Delta\phi_n(\mathbf{r})$ is the phase difference, with which the corresponding pressure components intersect at $\mathbf{r}$. As observed, the above radiation force is taken to consist of a cosine wave of duration D. In the immediate region surrounding the geometric focus, the primary component of the force is propagating perpendicularly from the face of the focused transducers. If N_h harmonics are considered for each ultrasound beam, then the modulated finite-amplitude radiation force at focus, can be written as

$$F^\Delta(\mathbf{r},t) = \sum_{n=1}^{N_h} F_n^\Delta(\mathbf{r},t) \tag{3}$$

where each harmonic component of the above summation is given by (2).

In response to the above finite-amplitude acoustic radiation force, modulated shear waves are generated that propagate away from the focal zone at multiples of the difference frequency Δf. In contrast to most shear-based elastography methods reported in the literature, these tone-burst generated shear waves can be shown to have a relatively narrow bandwidth (as mentioned

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