Ultrasonics xxx (2013) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

A hybrid FDTD-Rayleigh integral computational method for the simulation of the ultrasound measurement of proximal femur

Didier Cassereau [a,*], Pierre Nauleau [b], Aniss Bendjoudi [c], Jean-Gabriel Minonzio [b], Pascal Laugier [b], Emmanuel Bossy [c], Quentin Grimal [b]

[a] Laboratoire d'Imagerie Paramétrique, UPMC Univ Paris 06, CNRS UMR 7623, ESPCI ParisTech, 15 rue de l'Ecole de Médecine, 75006 Paris, France
[b] Laboratoire d'Imagerie Paramétrique, UPMC Univ Paris 06, CNRS UMR 7623, 15 rue de l'Ecole de Médecine, 75006 Paris, France
[c] Institut Langevin, CNRS UMR 7587, ESPCI ParisTech, INSERM ERL U979, Université Denis Diderot-Paris7, 1 rue Jussieu, 75005 Paris, France

A R T I C L E   I N F O

Article history:
Received 22 April 2013
Received in revised form 11 June 2013
Accepted 11 June 2013
Available online xxxx

Keywords:
Simulation
Semi-analytic
Femoral neck
Guided waves
DORT method

A B S T R A C T

The development of novel quantitative ultrasound (QUS) techniques to measure the hip is critically dependent on the possibility to simulate the ultrasound propagation. One specificity of hip QUS is that ultrasounds propagate through a large thickness of soft tissue, which can be modeled by a homogeneous fluid in a first approach. Finite difference time domain (FDTD) algorithms have been widely used to simulate QUS measurements but they are not adapted to simulate ultrasonic propagation over long distances in homogeneous media. In this paper, an hybrid numerical method is presented to simulate hip QUS measurements. A two-dimensional FDTD simulation in the vicinity of the bone is coupled to the semi-analytic calculation of the Rayleigh integral to compute the wave propagation between the probe and the bone. The method is used to simulate a setup dedicated to the measurement of circumferential guided waves in the cortical compartment of the femoral neck. The proposed approach is validated by comparison with a full FDTD simulation and with an experiment on a bone phantom. For a realistic QUS configuration, the computation time is estimated to be sixty times less with the hybrid method than with a full FDTD approach.

1. Introduction

Osteoporotic hip fractures are associated with a high mortality and high treatment costs [1]. Fracture risk is best predicted by site-matched measurement of areal bone mineral density (aBMD) with dual-energy X-ray absorptiometry [2]. However, aBMD alone is not sufficient to account for bone strength [3,4]. The development of a quantitative ultrasound (QUS) hip scanner [5,6] aims at providing an alternative to aBMD measurements. Because the cortical shell at the proximal femur determines a large part of bone strength [7,8], there is substantial interest in developing QUS measurements with improved sensitivity to cortical bone geometrical and material properties.

The propagation of waves around the circumference of the cortical shell of the femur neck (FN) was first evidenced with numerical simulations [9,10]. The interest for these waves was reinforced by the results of a pilot *in vitro* study [11] which suggested that there is a strong relationship between femur strength and the time-of-flight of the first arriving signal associated with waves traveling circumferentially. Furthermore, it is likely that several modes of circumferential guided waves can be measured in the cortical bone of the FN using multi-emitter and multi-receiver ultrasonic arrays and dedicated signal processing techniques such as the DORT method [12,13]. This method is suited to measure guided waves dispersion curves of a distant immersed object. These dispersion curves can in principle be processed to derive independent information on bone geometry and material properties.

The development of these novel QUS approaches using guided waves is critically dependent on the possibility to model and simulate the interaction of ultrasound with bone. It is particularly challenging to simulate QUS measurement of the FN because: (i) the shape of the FN is complex and (ii) the ultrasound probe is necessarily placed at a large distance from the bone, that is, the ultrasound beam must travel through a large thickness of soft tissue. In order to model the shape of the FN, numerical methods based on a discretization of space like finite elements or finite difference methods must be used. However these methods are in general not adapted to simulate the propagation over large distances (i.e. large number of wavelengths) in homogeneous or quasi-homogeneous media (soft tissue). To avoid the accumulation of numerical errors along the wave path, a fine resolution is needed, which in

* Corresponding author.
E-mail address: didier.cassereau@upmc.fr (D. Cassereau).

0041-624X/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ultras.2013.06.004

general results in prohibitively high computational costs. Also, in many cases it is important to keep the computation time as low as possible because of the necessity to repeat the simulations for a given configuration to account for multiple emissions when multi-emitter arrays are used. Some signal processing methods, e.g. the DORT method, based on the analysis of the response matrix of the wave guide, require to perform several successive emissions.

The aim of this paper is to present the implementation of an hybrid numerical method to simulate the QUS measurements at the FN. The method consists in coupling a finite difference time domain (FDTD) simulation to compute the ultrasound wave field in the vicinity of the FN and a semi-analytic technique which evaluates the Rayleigh integral to compute the wave propagation between the ultrasonic array and the FDTD domain. In this paper we consider the simulation of a set-up which is dedicated to the measurement of circumferential guided waves, but the proposed hybrid method is not restricted to this configuration.

2. Configuration and model

We consider the modeling of a measurement set-up which aims at recording the wave field scattered by the cortical bone shell of the FN (Fig. 1) [13]. The transducer is a phased array of 128 independent elements working in the transmit/receive mode. It is cylindrically pre-focused in order to concentrate the beam energy in a plane perpendicular to the FN axis, which is placed at the focal distance. A similar setup with a monoelement probe was used previously to measure femurs *in vitro* [11].

The bone is supposed to be immersed in a homogeneous fluid with the properties of water which represents soft tissue and coupling medium between the array and the skin. Cortical bone is modeled as an elastic solid. Attenuation in cortical bone and water was not considered. Trabecular bone and marrow in the cortical shell are not modeled for the sake of simplicity, i.e. the interior of the shell is modeled as void.

For the purpose of simulating the measurement of the ultrasound field scattered by the bone shell, the physical space is split in two domains: $\mathcal{A}$ is a rectangular area in the vicinity of the bone; $\mathcal{B}$ designates the remaining of space and in particular the space between the transducer and domain $\mathcal{A}$.

The model is three-dimensional in $\mathcal{B}$ and two-dimensional in $\mathcal{A}$. The two-dimensional approximation consists in neglecting the diffraction of the incident beam in the **z**-direction, which is motivated by the fact that the diffracting object is a cylinder of axis **z** and that the probe is pre-focused in a plane perpendicular to the **z** axis. The focal width at −6 dB is 4 mm and the depth of field is 35 mm: in this region, which includes the coupling line Σ and the cylinder, the incident wave can be considered to be plane.

3. Hybrid FDTD/Rayleigh integral computational method

3.1. FDTD

In domain $\mathcal{A}$ the elastodynamics equations

$$\rho(\mathbf{x})\frac{\partial v_i}{\partial t}(\mathbf{x},t) = \sum_{j=1}^{d}\frac{\partial \sigma_{ij}}{\partial x_j}(\mathbf{x},t), \quad (1)$$

$$\frac{\partial \sigma_{ij}}{\partial t}(\mathbf{x},t) = \sum_{j=1}^{d}\sum_{i=1}^{d} c_{ijkl}(\mathbf{x})\ \frac{\partial v_k}{\partial x_l}(\mathbf{x},t), \quad (2)$$

are discretized and the wave field is computed with the FDTD method implemented in Simsonic software [14]. This sofware implements Virieux's scheme [15,16]. In Eqs. (1) and (2), ρ is the density, v_i are the components of the particle velocity, c_{ijkl} are the components of the stiffness tensor, and σ_{ij} are the components of the stress tensor. These quantities are defined at each point **x** of the FDTD grid. The spatial domain is discretized over a cartesian mesh with an isotropic grid step Δx. The temporal step Δt for time discretization must satisfy the stability condition (usually known as the Courant, Friedrichs and Levy condition)

$$\Delta t \leqslant \frac{1}{\sqrt{2}} \times \frac{\Delta x}{c_{max}},$$

where c_{max} is the largest propagation velocity in domain $\mathcal{A}$. The accuracy of the FDTD numerical computation depends on the choice of Δx. This choice results of a compromise between accuracy, computation time, and available resources.

3.2. Rayleigh integral

The propagation of waves in domain $\mathcal{B}$, in particular between the transducer and domain $\mathcal{A}$, can be treated in the framework of the standard impulse diffraction formulation in a homogeneous fluid. In the case of a planar vibrating surface mounted in an infinitely rigid baffle, the transient diffracted beam can be calculated using Green's functions approach [17,18]. The velocity potential at point **r** due to each element of the array writes

$$\Phi(\mathbf{r},t) = \varphi(t) \underset{t}{*} h(\mathbf{r},t),$$

where $*/t$ denotes the temporal convolution, $\varphi(t)$ is the excitation function and $h(\mathbf{r},t)$ is the impulse Rayleigh integral defined by

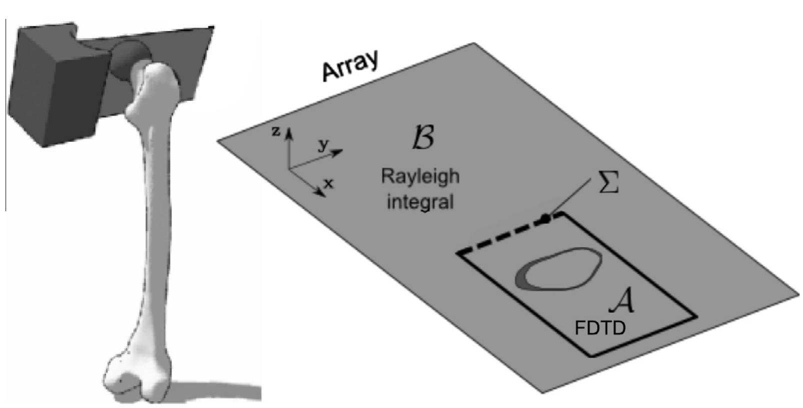

Fig. 1. Representation of the configuration for the measurement of the wave field backscattered by the femoral neck cortical shell (left). The measurement is performed in the focal plane of the pre-focused array which intersects a cross-section of the neck (gray area). $\mathcal{A}$ designates an area in the vicinity of the bone cross-section where the wave field is computed with FDTD; $\mathcal{B}$ designates a homogeneous fluid domain in which the ultrasound propagation between the array and the coupling line Σ is calculated by evaluating the Rayleigh integral (right).

Please cite this article in press as: D. Cassereau et al., A hybrid FDTD-Rayleigh integral computational method for the simulation of the ultrasound measurement of proximal femur, Ultrasonics (2013), http://dx.doi.org/10.1016/j.ultras.2013.06.004