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Band gap structures in underwater screens of periodically spaced porous plates

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ABSTRACT

Acoustic properties of different periodic structures composed of alternating fluid and fluid-saturated porous layers obeying Biot's theory are investigated. At first, the network of modes and the transmission coefficients of finite structures of six plates are studied in the frequency-angle of incidence plane. It is shown that the network of modes concentrates in localized domains of the plane where the transmission coefficients will take the greatest values. With this minimum of six plates, the structures exhibit the main features as for structures containing more plates, especially those with an infinite number of plates. Then, considering infinite structures the band gap calculations are led using the Bloch–Floquet theorem. The evanescent and propagative zones in the frequency-angle of incidence plane are determined. What is proposed here is a class of underwater porous screens that exhibits band gaps extending over great angular domains and enlarging in the frequency domain when the pores at the interfaces of the porous plates are sealed. The effect of porosity on the band gaps is also investigated.

1. Introduction

In the last decades, the analysis of layered geological structures together with the design of composite materials having specific properties led to several papers. Schoenberg and Sen [1] applied the method of transfer matrices [2–4] to a periodically layered acoustic half-space to study the pass and stop bands. Rousseau [5] applying the same method studied Bloch and Floquet [6,7] wave properties of periodic structures of elastic solid and fluid layers. Later, Potel and de Belleval [8] extended the study to periodically layered anisotropic media. Media considered in those works are single phase media; the waves that propagate in each medium being neither dispersive nor attenuated.

Heterogeneous materials made up of periodic arrays of inclusions in a fluid or an elastic medium have also retained the attention because of their capacity to create pass and stop bands similar to those encountered in layered media. Band gaps are of great interest in various domains such as the design of silent blocks [9]. To this end, some authors presented methods to obtain large band gaps [10,11].

In this paper, the band gaps created by underwater systems made up of alternating porous plates and liquid layers are studied. In the fluid saturated porous medium obeying Biot's theory [12,13], two longitudinal waves and one shear wave propagate that are all dispersive and attenuated. One of the longitudinal wave is referred to as the 'fast wave' while the remaining is called the 'slow wave'. All these waves are dispersive and attenuated, contrary to waves propagating in single phase elastic media. When the porous medium is reduced to a plate of constant thickness, Rasolofosaon [14] showed that the slow wave generation depends on the boundary conditions. For open pore conditions at the interfaces separating the porous medium from the surrounding fluid, the contribution of the slow wave to the modal waves is not negligible. At the opposite, the sealing of the pores at the interfaces provokes the disappearance of the slow wave which becomes diffusive at all frequencies. As it will be shown below, for the formation of acoustic band gaps with systems of several plates, the sealing case is the more interesting among the two kinds of boundary conditions in so far as one is interested chiefly by the creation of the largest band gaps. To this end, approximate expressions for reflection and transmission studies of porous plates with sealed pores are briefly recalled in Section 2.

In Section 3, a comparison between exact and approximate results obtained for reflection and transmission coefficients of systems with six equidistant plates is presented. This number of six plates, although not rigorously fixed, allows to exhibit the main features as for structures containing more plates, especially those with an infinite number of plates. Section 4 deals with the band gap phenomena for infinite periodic systems of alternating fluid and porous plates. Both cases of sealed and open pores at the interfaces are presented. Since one is concerned here with absorbing media and dispersive waves in a porous medium, the methods and approximations based upon the assumption of perfect media

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as in Ref. [5]no longer apply. The equation used for the computation of the Bloch–Floquet wavenumber is similar to that used by Heckl [9] to study sound propagation in periodically arranged cylindrical tubes and by Ashcroft and Mermin [15] in solid state physics. It is used to find the expression of first critical frequency at which the first band gap will occur at normal incidence. The knowledge of such frequency brings some information on the pertinent parameters to modify for lowering it. In Section 5, the effects of a decrease or an increase of the porosity on the band gaps location are studied.

2. Background

Let us consider a plane parallel poroelastic plate with a thickness d, saturated and surrounded by water. The faces are identically surfaced (either open pores or sealed pores). One way to obtain plates with sealed pores is described in [16]. Let r and t be the reflection coefficient (back into the fluid) and transmission coefficient (at the rear) when a plane monochromatic wave with angular frequency ω hits the plate at an angle θ with respect to the normal to the faces. They can be expressed as in [17] and then computed. However, because of the greatest performances observed in the separation the Lamb modes into two sets, symmetric (S) and antisymmetric (A) ones, the transition terms

$$\mathcal{T}_S^{(1)} = \frac{1}{2i}(r + t - 1), \quad \mathcal{T}_A^{(1)} = \frac{1}{2i}(r - t - 1), \tag{1}$$

are introduced here. They are better tools than reflection and transmission coefficients for both experimental and theoretical studies. It must be noticed that the expressions obtained for the exact reflection/transmission coefficients, see Eqs. (33–38) and (41–46) of [17] are not easy to handle. Consequently, for systems involving the association of several plates it is more convenient to consider approximate formulas. For single plates with sealed pores, that are the basis of the systems studied below the following approximations are recalled

$$\mathcal{T}_{S\ sealed}^{(1)} \simeq \frac{\underset{\sim 1}{\tau}}{C_{S1} - i\,\underset{\sim 1}{\tau}}, \quad \mathcal{T}_{A\ sealed}^{(1)} \simeq \frac{\underset{\sim 1}{\tau}}{C_{A1} + i\,\underset{\sim 1}{\tau}} \tag{2}$$

Details on the terms C_{A1}, C_{S1} and $\underset{\sim 1}{\tau}$ are given in the Appendix (see also Eqs. (63) and (66) of [17]). The subscript 1 recalls that only the fast wave is involved whereas the superscript (1) indicates that only one plate is involved. The slow wave (terms with the subscript 2) normally present in a Biot's medium, becomes effective only from the second order of the approximation neglected here in accordance with the fact proved experimentally [14,16] that the sealing of the pores at the boundaries reduces drastically the amplitude of the transmitted slow wave. The transition terms, Eqs. (2), allow the description of modes similar to the well known (A) and (S) modes of immersed elastic plates since only the fast and the shear waves are involved. Note that the combination of Eqs. (1) and (2)yields r and t in terms of $\mathcal{T}_S^{(1)}$ and $\mathcal{T}_A^{(1)}$.

Since the waves that travel in the porous medium are dispersive, the critical angles, θ_{c1} for the fast wave and θ_{ct} for the shear wave are frequency dependent. The use of the parameters of Table 1 shows that the critical angles decrease slightly when the frequency increases lying however in the bounds $24°8 \le \theta_{c1} \leqslant 26°$ and $45°2 \le \theta_{c1} \leqslant 48°2$. There is no critical angle for the slow wave.

3. Finite periodic systems of plates

Several plane identical plates are periodically spaced in a liquid (water) of infinite extent in which sound propagates with the velocity $c_0 = \sqrt{K_0/\rho_0}$; K_0 is the bulk modulus and ρ_0 the density (see Table 1). Two consecutive plates enclose a liquid layer of thickness D; the spatial period is then equal to $d + D$. When a wave traveling from the surrounding liquid impinges one of the outer plates at the angle of incidence θ, the system of $N \geqslant 2$ plates gives rise to reflected waves, transmitted waves and to forward and backward waves between two consecutive plates. The reflection and transmission coefficients r_N and t_N by the system of N plates can be calculated using the following recurrence formulas

$$r_N = r_{N-1} + \frac{t_{N-1}^2 r e^{-i\varphi_r}}{1 - r\, r_{N-1} e^{-i\varphi_r}}, \tag{3}$$

$$t_N = \frac{t\, t_{N-1} e^{-i\varphi_r/2}}{1 - r r_{N-1} e^{-i\varphi_r}}, \tag{4}$$

where $\varphi_r = 2k_0 D\cos\theta$ is a phase difference. The factor $k_0 = \omega/c_0$ represents the wavenumber in the liquid surrounding the plates. To have an idea of the modes produced by a system with $N = 6$ plates, and especially of the differences generated by the boundary conditions (open or sealed pores), we use the transition terms $\mathcal{T}_S^{(6)} = (r_6 + t_6 - 1)/2i$.

Table 1
QF20 and saturating water parameters (all data from Johnson et al. [16] except pore radius estimated by us).

Bulk modulus of grains	K_r (Pa)	36.6×10^9
Dried frame bulk modulus	K_b (Pa)	9.47×10^9
Dried frame shear modulus	μ (Pa)	7.63×10^9
Solid density	ρ_s (kg m^{-3})	2760
Bulk modulus of water	K_0 (Pa)	2.22×10^9
Density of water	ρ_0 (kg m^{-3})	1000
Viscosity of saturating water	η (kg m^{-1} s^{-1})	1.14×10^{-3}
Porosity	β	0.402
Permeability	k (m^2)	1.68×10^{-11}
Pores radius	a_p (m)	3.26×10^{-5}
Tortuosity	α	1.89

3.1. Sealed pores

A spacing $D = 1$ *cm* is considered between plates (of thickness $d = 0.5$ *cm*). The modulus of the transition terms $\mathcal{T}_S^{(6)}$ are presented in the $(fd, \sin\theta)$ plane, in Fig. 1 for the exact case, in Fig. 2 for the approximate case calculated from Eq. (2). The very good agreement between the two networks of modes (white lines) legitimates the use of the approximations given by Eq. (2) for computing transition terms of structures with a greater number of plates and even for periodic structures. In comparison, Fig. 3 which represents the

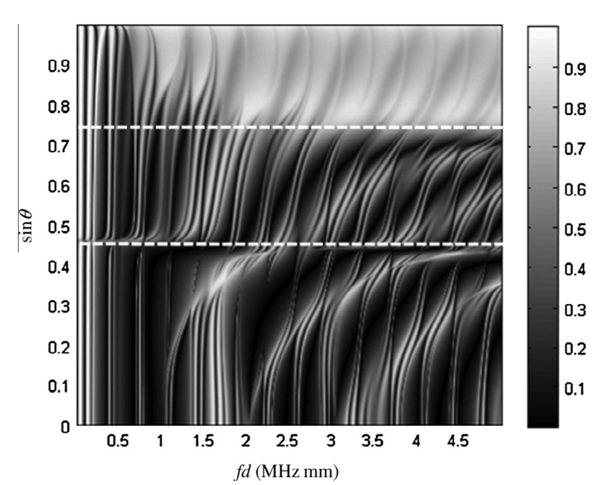

Fig. 1. Sealed pore condition. The modulus of the exact $\mathcal{T}_S^{(6)}$ with a spacing $D = 1$ *cm* between plates. The lightest zones correspond to the highest magnitudes. The positions of k_x corresponding to critical angles are indicated by dot horizontal lines.