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Stable bubble oscillations beyond Blake's critical threshold

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ABSTRACT

The equilibrium radius of a single spherical bubble containing both non-condensable gas and vapor is determined by the mechanical balance at the bubble interface. This expression highlights the fact that decreasing the ambient pressure below the so called Blake's critical threshold, the bubble has no equilibrium state at all. In the last decade many authors have tried to find evidence for the existence of stable bubble oscillation under harmonic forcing in this regime, that is, they have tried to stabilize the bubble motion applying ultrasonic radiation on the bubble. The available numerical results provide only partial proof for the existence as they are usually based on linearized or weakly nonlinear (higher order approximation) bubble models. Here, based on numerical techniques of the modern nonlinear and bifurcation theory, the existence of stable bubble motion has been proven without any restrictions in nonlinearities. Although the model, applied in this paper, is the rather simple Rayleigh–Plesset equation, the presented technique can be extended to more complex bubble models easily.

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1. Introduction

The equilibrium radius of a gas- and/or vapor-filled spherical bubble is characterized by the well-known static mechanical equilibrium at the bubble interface. For a given mass of gas m_G and ambient temperature T_∞ it is written as

$$0 = p_V - P_\infty + p_{go}\left(\frac{R_o}{R_E}\right)^{3n} - \frac{2\sigma}{R_E}, \qquad (1)$$

where R_E is the equilibrium bubble radius, $\sigma(T_\infty)$ is the surface tension, $p_V(T_\infty)$ is the vapor pressure and P_∞ is the static ambient pressure far away from the bubble. The reference pressure and radius and the exponent of the polytropic state of change are p_{go}, R_o and n, respectively. The reference quantities determine the mass of gas within the bubble:

$$m_G = \frac{4 p_{go} R_o^3 \pi}{3 \Re T_\infty}, \qquad (2)$$

where $\Re$ is the specific gas constant. Observe that Eq. (2) corresponds to equilibrium conditions, thus, the constant ambient temperature T_∞ does not imply isothermal gas behavior.

Typical equilibrium bubble radius curves are presented in Fig. 1 for water at $T_\infty = 37\,^\circ$C. The stable R_E^s and unstable R_E^u radii are marked by the solid and dashed lines, respectively. If the bubble interior contains non-condensable gas beside the vapor, that is, $m_G > 0$ than a turning point appears in the equilibrium radius curve called Blake's critical threshold, which first identified by Blake [6] and later on Neppiras and Noltingk [23], see the back dot in Fig. 1. Throughout this paper the equilibrium radius at this critical point will be referred to as critical radius R_C. It is important to emphasize that beyond Blake's threshold there is no equilibrium state of the bubble. In the absence of foreign gas ($m_G = 0$), however, no stable equilibrium exists at all, resulting in an infinite growth or a total collapse of the vapor bubble for all values of the ambient pressure. The above-described graphical representation was used by Daily and Johnson [8] and a thorough discussion of the equilibrium state of a bubble can also be found in Brennen [7].

Although the above description excludes the existence of stable vapor bubbles, the experiment of Marston and Greene [20] reported the observation of stable microscopic vapor bubbles under a harmonically varying pressure field, discovering the fact that ultrasonic radiation may stabilize the bubble motion. The authors also concluded that the stability may be influenced by the previously found second (evaporation–condensation) resonance. Beside the so-called Minnaert primary resonant frequency [21] this second resonance frequency was first reported by Neppiras and Finch [22] in vapor bubbles for a given equilibrium radius. Their observation was supported by other authors, see Wang [29] or Wang [30]. The discovery of this phenomenon had triggered an increasing attention of the researchers on the subject. Several versions of analytical approximations for this second resonance were proposed, with different degrees of complexity, by Patel et al. [27], Marston [19], Hsieh [16], Akulichev [1], Alekseev [2] and many others. Since these studies are based on first or second order approximations, their results are limited only to small amplitude oscillations.

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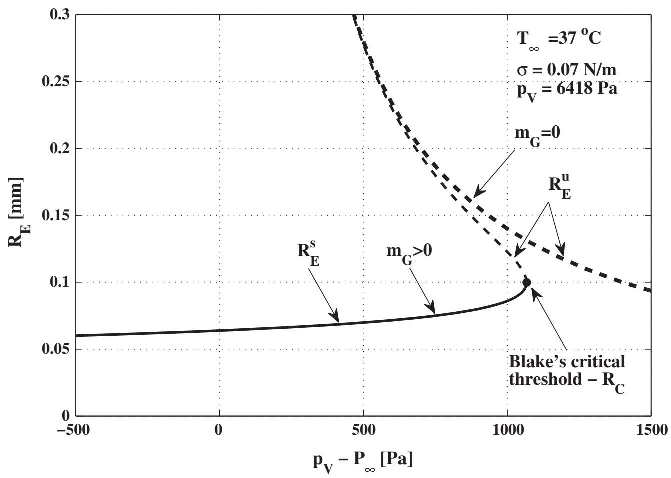

Fig. 1. Typical equilibrium bubble radius curves for water at $T_\infty = 37\ ^\circ$C. The solid and dashed lines are the stable and unstable equilibrium radius curves, respectively. The thick dashed curve corresponds to pure vapor bubbles, whereas in the case of the thin curve the bubble interior contains both vapor and foreign non-condensable gas. The black dot denotes the so-called Blake's critical threshold.

These results encouraged Hao and Prosperetti [12] to solve a relatively complex bubble model numerically, without any linearization or other approximations, to verify the predictions of the linear theories and try to achieve stable bubble oscillations under harmonic forcing. Unfortunately, they could not find any stable oscillation even after thousands of cycles. A thorough analysis of Gumerov [10] provided a detailed explanation on the stability of vapor bubbles and revealed that in the parameter range applied by Hao and Prosperetti [12] there is no stable bubble oscillation.

The valuable results of Gumerov [10] proved that may be possible to stabilize a vapor bubble with harmonic forcing even if there is no any stable equilibrium or any equilibrium solution of the unexcited system. However, the question of this stabilization mechanism for large amplitude (nonlinear) oscillations is still open due to the applied third order approximation. The exponentially increasing computational resources, the rapidly developing and spreading nonlinear theory and its numerical techniques could help to overcome the problem. Still, the majority of the corresponding papers of the topic use less sophisticated techniques, such as the so-called brute force method, hence often miss the chance to find stable solutions, see again Hao and Prosperetti [12]. Even if some authors make use of the benefit of the advanced numerics the applied parameter values are simply out of the range of interest, see Parlitz et al. [25], Simon et al. [28], Behnia et al. [3–5]. In these cases the bubble contains non-condensable gas ($m_G > 0$) and the ambient pressure is so high that stable bubble oscillations always exist (far below Blake's critical threshold). One of the most recent review paper summarizing the achievements of nonlinear dynamics on bubble oscillations is written by Lauterborn and Kurz [18].

The main aim of this study is to prove that periodic forcing can stabilize the bubble motion even if there is no stable bubble equilibrium or any equilibrium radius of the unexcited system. Although the employed bubble model is rather simple, the well-known Rayleigh–Plesset equation, there is no restriction (linearization or other reduced order modelling) on our numerical technique. This means that the results are valid also for large amplitude oscillations, of course within the accuracy of the physical modelling. The surrounding of the bubble is pure water and the bubble content is a mixture of water vapor and non-condensable gas ($m_G > 0$). Therefore, the region where stable equilibrium does not exist lies beyond Blake's critical threshold, see Fig. 1. The procedure described here to find stable solutions can be easily applied to more complex models and for pure vapor bubbles ($m_G = 0$).

2. Mathematical model

The model describing the evolution of the bubble radius in time is rather simple. Integrating the one dimensional Navier–Stokes equation (incompressible liquid) from the bubble radius $R(t)$ to infinity making use of the equation of continuity, one can obtain the equation

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_L}(p_L - p_\infty) \tag{3}$$

known as the Rayleigh–Plesset equation which is a second order ordinary differential equation. Here, ρ_L is the density of the liquid, p_L is the pressure at the bubble wall in the liquid domain and p_∞ is the pressure far away from the bubble consisting of static and periodic components written as

$$p_\infty(t) = P_\infty + p_A \sin(\omega t), \tag{4}$$

where P_∞ is the ambient pressure, p_A and ω is the pressure amplitude and angular frequency of the excitation, respectively. The bubble interior contains both vapor and non-condensable gas assuming ideal gas behavior. The pressure of this ideal mixture is composed by the sum of the partial pressure of vapor p_V and the gas p_G. The connection between the pressure inside and outside the bubble at the bubble interface is expressed by the mechanical balance

$$p_G + p_V = p_L + \frac{2\sigma}{R} + 4\mu_L\frac{\dot{R}}{R}, \tag{5}$$

where σ is the surface tension and μ_L is the liquid dynamic viscosity. The vapor pressure inside the bubble is constant but its value depends on the ambient temperature T_∞ while the gas content obeys a simple polytropic relationship

$$p_G = p_{go}\left(\frac{R_o}{R}\right)^{3n} \tag{6}$$

with a polytropic exponent n. Although n can vary between 1 and 1.4 (isothermal and adiabatic behavior), this study follows the concept of Lauterborn and Kurz [18] and $n = 1.4$ was applied. The reference pressure p_{go} and radius R_o determine the mass of gas inside the bubble, see Eq. (2). The material properties of the liquid domain (water) and the water vapor inside the bubble were computed by means of the Haar–Gallagher–Kell equation of state (see [11]) at T_∞ and P_∞, whose values are discussed in the next subsection.

2.1. Parameters and their transformations

Before presenting any specific computations, let us summarize the parameters needed to specify in the model. This paper follows the idea dealing with the large number of parameters, first proposed by Hegedűs et al. [13], based on the fact that the material properties of a pure substance depend on the pressure P_∞ and temperature T_∞. Regarding these two quantities as independent parameters, all the material properties can be determined, hence, the number of parameters can be significantly reduced, especially in complex bubble models. Beside the ambient properties a further parameter for the description of the bubble size is still needed. This can be, for instance, the mass of gas within the bubble (m_G), the equilibrium bubble radius (R_E) or the critical radius (R_C, see the investigation below). Finally, in the presence of harmonic forcing the pressure amplitude p_A and angular frequency ω are also required.

First, let us discuss the description of the bubble size. The mass of gas m_G is not an illustrative measure. The equilibrium radius R_E

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