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Interaction of Lamb modes with an inclusion

G. Shkerdin [1], C. Glorieux *

Laboratorium voor Akoestiek en Thermische Fysica, Departement Natuurkunde, K.U. Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium

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ABSTRACT

The interaction of Lamb modes propagating in a steel plate containing a thin inclusion is analyzed for cases where the inclusion material has elastic parameters similar to the ones of the plate, and where the inclusion is in perfect mechanical contact with the surrounding plate material. A modal decomposition method is used to calculate the conversion of an incident Lamb mode to other modes. Hence, the influence of the type of incident mode, of the location and geometry of the inclusion, and of the elastic parameters of the inclusion and plate material on the mode conversion coefficients is analyzed. Besides the expected increase of the conversion efficiency with increasing cross section of the inclusion, it is found that due to reasons of symmetry, the presence of an inclusion leads to an efficient conversion of an incident S0 mode into reflected and transmitted A0 modes, unless the inclusion is located very close to the plate center. On the other hand, the conversion efficiency of an incident A0 mode into a reflected A0 mode is found to be strongly dependent on the depth of the inclusion, this conversion even disappearing for some location depths. For the cases studied, the inclusion location dependence of the mode conversion seems to be correlated with the normal profile of the longitudinal normal stress component $\sigma_{yy}(y)$. As intuitively expected, the mode conversion efficiency increases with the mismatch of an acoustic impedance like factor between the uniform plate and the inclusion region.

1. Introduction

Motivated by applications for non-destructive characterization of materials, the interaction of acoustic waves with defects inside materials has been widely studied during the past years (see, e.g., Refs. [1–9]). Among the wide variety of defects in plate-like materials, inclusions, i.e. regions inside a plate with density and elastic properties different from these of the plate material, represent an important class. For example, cracks and delaminations filled with air or dirt can be considered as inclusions, and their timely detection by means of non-destructive testing techniques can be very crucial for reasons of safety or performance in a production line or transport container. Exciting and detecting Lamb modes offers a quite convenient way to remotely monitor the state of large regions in a plate of interest (e.g. the wall of a container or pipeline, the tail boom of a helicopter, or the walls of an airplane cabin), since Lamb waves easily scatter on acoustic heterogeneities. The interaction of Lamb modes with different kinds of delaminations (see, e.g., Refs. [1–5]) and notches or holes in plates (see, e.g., Refs. [6–9]) has therefore attracted quite some attention.

Due to their wavelength typically being of the order of the plate thickness, Lamb modes are very dispersive and the possible number of propagating symmetric and antisymmetric modes strongly depends on frequency. When a Lamb mode encounters a heterogeneity, then part of its energy is channelling to reflected and transmitted modes of different types. Till now, the interaction of Lamb modes with defects was mostly analyzed for an acoustically thin plate, in which case only the lowest order symmetric and antisymmetric Lamb modes can propagate. The existence of only a few modes in a detected displacement signal allows to easily discriminate between different (defect-independent) excited modes on one hand, and defect-induced modes on the other hand. The propagation of Lamb modes through a thin absorptive bi-layer with delamination was theoretically analyzed in Ref. [10], where it was shown that, although the incident energy flow conversion into reflected and another transmitted modes is very small, delaminations can be experimentally assessed analyzing the tangential dependence of the normal displacement along bi-layer surfaces. The amplitude of the normal displacement at the plate interfaces is frequently measured to analyze propagating acoustic waves (see, e.g., Refs. [11,12]).

Notches or holes filled with air or soft dirt represent a subset of inclusions for which the difference between the inclusion and plate material parameters is very large. As a result, the acoustic wave diffraction on these defects is typically quite strong, leading to a considerable fraction of incident energy being reflected, and to

* Corresponding author.
E-mail address: christ.glorieux@fys.kuleuven.be (C. Glorieux).
[1] Address: Institute of Radio Engineering and Electronics of Russian Academy of Sciences, Vvedensky sq., 1, Fryazino, Moscow Region, Russia.

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significant mode conversion in transmission. As a result, notches and holes can be quite easily detected by analyzing the modal content in reflected or transmitted displacement signals detected along the plate of interest. Quite often though, inclusions are very thin and the elastic parameters of the material filling an inclusion can be quite similar to the ones of the surrounding material, resulting in small and thus difficult to detect influences on the wave propagation. The present paper is concerned with the latter situation and aims at quantifying the mode conversion coefficients in the case of weak interaction, and at gaining insight in the factors determining the sensitivity of different types of Lamb modes to the geometrical and elastic characteristics of the inclusion and its surroundings. Although our interest here is mainly oriented towards the detection limit, in terms of trends, aspects of symmetry and order of magnitude of effects of inclusions on the signal, the results can easily be extrapolated to defects with higher contrast.

The calculation is based on a modal decomposition method that was introduced earlier for monolithic plates and for bi-layers (see Refs. [13,14]). The generalization of the method to the case Lamb mode interaction with an inclusion in a plate is concisely summarized in Section 2. In Section 3, the model is applied to the systematic study of the dependence of the conversion of an incident mode on the parameters of the inclusion. The mode conversion is assumed to be monitored via the normal displacement amplitude at the plate surfaces. In this part, also the effect on the numerical calculation accuracy of truncating the infinite system of equations arising from implementing boundary conditions on the infinite set of modes is analyzed.

2. Modal decomposition method in a plate with an inclusion

The two-dimensional geometry (we consider cases where there are only displacement and stress components in the YZ plane) under consideration is shown schematically in Fig. 1. An incident right-going sinusoidal Lamb wave with angular frequency ω and known magnitude propagates along the z-direction in an isotropic plate with thickness d. The plate surfaces at $y = 0,\ y = d$ are everywhere stress-free. A rectangular inclusion, located in the intersection between the regions $d_1 < y < d_2$ and $0 < z < L$, is assumed to be in perfect mechanical contact with the surrounding plate material, resulting in permanent continuity of the mechanical displacement components and of the proper stress tensor components at the interfaces $y = d_1,\ y = d_2$ for $0 \leqslant z \leqslant L$ and the interfaces $z = 0,\ z = L$ for $d_1 \leqslant y \leqslant d_2$.

The method to calculate the acoustic field resulting from the incident wave, and from converted waves that are generated due to its interaction with the inclusion, is based on an expansion of this field in the Lamb eigenmodes in each region of the composite plate, and on the use of boundary conditions at the vertical planes $z = 0$ and $z = L$ to determine the expansion coefficients. Since this method is described in detail in Refs. [10,13] for the case of delaminations, here we just give the concise description of its generalization for an inclusion.

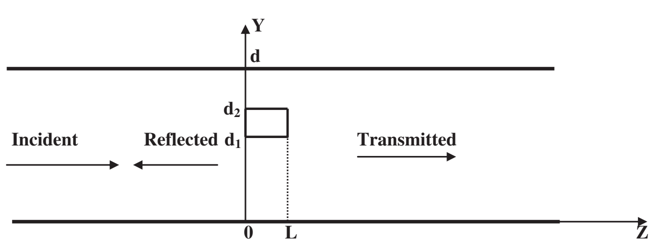

Fig. 1. Schematic view of Lamb wave incident on an inclusion in a plate.

The components of the stress tensor S^n_{ik} and the mechanical displacement vector $U^n_i (i = y, z$ and $k = y, z)$ of every (nth) right-going Lamb mode are given by:

$$(S^n_{ik}, U^n_i) = (\sigma^n_{ik}, u^n_i)Exp(iq_n z - i\omega t) + (\sigma^n_{ik}, u^n_i)^* Exp(i\omega t - iq^*_n z) \quad (1)$$

where the asterisk symbol refers to complex conjugation and the functions $\sigma^n_{ik}(y)$ and $u^n_i(y)$ depend on y, the coordinate normal to the plate, which is taken to be zero at the bottom of the plate. Every Lamb mode wave number q_n is satisfying the Lamb dispersion equation (see Refs. [14,15] for the monolithic plate region and see Appendix A for the threefold composite inclusion region $0 < z < L$). The normal depth profiles (y-direction) of the dynamic variables $S^n_{ik},\ U^n_i,\ \sigma^n_{ik},\ u^n_i$ of the different modes (index n) are all different, but every mode satisfies the wave equation, the stress–strain relations and the stress-free conditions at $y = 0$ and $y = d$. The Lamb modes in the composite inclusion region ($0 < z < L$, see Fig. 1) satisfy additional continuity conditions reflecting perfect mechanical contact at the tangential interfaces $y = d_1,\ d_2$. All Lamb modes are mutually orthogonal (see e.g. Eq. (2) in Ref. [10]).

The expansion of the mechanical displacement vector for the transmitted acoustic waves $\vec{u}^r(z > L)$ can be written as:

$$\vec{U}^{tr} = \sum_n C^{tr}_n \vec{u}^n_{tr} Exp(i(q_n(z - L) - \omega t)) \quad (2)$$

with C^{tr}_n the expansion coefficients for transmitted acoustic waves, which are to be determined. The expression is analogous for the reflected acoustic waves ($z < 0$):

$$\vec{U}^{ref} = \sum_n C^{ref}_n \vec{u}^n_{ref} Exp(-i(q_n z + \omega t)) \quad (3)$$

The expansion of the mechanical displacement vector for right-going acoustic waves in the region $0 < z < L$ (indicated by index '1') containing the inclusion, $\vec{U}^{tr,1}$, is the same as the one of in Eq. (2), provided the following substitutions: $C^{tr}_n \to C^{tr,1}_n,\ \vec{u}^n_{tr} \to \vec{u}^n_{tr,1},\ q_n \to q_{n,1}$ and $L \to 0$. The expansion for the left-going acoustic waves $\vec{U}^{ref,1}$ in this region is given by Eq. (3), with analogous substitutions and $z \to z - L$.

Applying the boundary conditions expressing perfect mechanical contact at the normal interfaces $z = 0,\ z = L$ and using the orthogonality condition as in Ref. [10] an infinite system of linear equations Eqs. (4)–(7) in the unknown mode coefficients $C^{tr,ref}_n,\ C^{tr,1}_n$ and $C^{ref,1}_n$ can be derived:

$$A(n)C^{tr}_n = \sum_m \left(C^{tr,1}_m e^{iq_{m,1}L}(K_{n,m} + I_{n,m}) + C^{ref,1}_m (I_{n,m} - K_{n,m}) \right) \quad (4)$$

$$A(n)C^{ref}_n = \sum_m \left(C^{ref,1}_m e^{iq_{m,1}L}(K_{n,m} + I_{n,m}) + C^{tr,1}_m (I_{n,m} - K_{n,m}) \right) \quad (5)$$

$$A_1(m)C^{tr,1}_m = \sum_n \left(C^{in}_n (K_{n,m} + I_{n,m}) + C^{ref}_n (K_{n,m} - I_{n,m}) \right) \quad (6)$$

$$A_1(m)C^{ref,1}_m = \sum_n C^{tr}_n (K_{n,m} - I_{n,m})) \quad (7)$$

where $I_{n,m} = \int_0^d dy \left(\sigma^n_{yz,tr} u^m_{y,tr,1} - u^n_{z,tr} \sigma^m_{zz,tr,1} \right),\ K_{n,m} = \int_0^d dy \left(\sigma^m_{yz,tr,1} u^n_{y,tr} - u^m_{z,tr,1} \sigma^n_{zz,tr} \right)$ can be considered as overlap integrals between the nth and mth modes of the adjacent (monolithic and threefold) regions, with $A(n),\ A_1(m)$ normalization factors.

3. Mechanical displacement calculations in a plate with an inclusion

3.1. Parameters used for numerical implementation of the model

In this section, we implement the theoretical model for calculating mode conversion coefficients for the case of a non-absorptive steel plate that contains an inclusion with density, shear velocity