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A study of time harmonic guided Lamb waves and their caustics in composite plates

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ABSTRACT

Spatial steady-state Lamb wave propagation in an anisotropic composite plate excited by harmonic surface sources is modeled using a Green's matrix representation in a frequency-wavenumber domain. An approach based on a residue integration technique for two dimensional wavenumber integrals for the computation of displacements outside an excitation source is presented in this paper. In the far-field zone of the excitation source, the method of stationary phase is used, which gives an asymptotic expansion of the displacement vector as a sum of cylindrical waves. Near caustic directions, a far-field solution is computed in terms of Airy functions. The results obtained applying residue integration technique and asymptotic expansion are found to be coinciding with the results of the computation by using the adaptive quadratures. Moreover, these approaches agree well with experimental data. Then, the advantages and disadvantages of the various methods applied for modeling of Lamb wave propagation are discussed in this paper. Focussing and other properties of Lamb waves are studied using numerical examples.

1. Introduction

In recent years there has been an obvious tendency to widely use composites in different branches of engineering and industry. The basic advantage of composites is their higher strength relative to mass in comparison with metals. Moreover, composites have higher corrosion stability. At the same time, composites are more sensitive to impact actions, which can cause damages in the form of cracks or delaminations which could potentially result in destruction. Defects in composite materials are promised to be identified applying the methods based on the use of elastic Lamb waves [1] propagating in structures under surface excitation [2].

Studies of surface-excited waves in multilayered structures have received high attention in the last 20 years. Waveguides are frequently modeled in two dimensions [3] for understanding many aspects of wave propagation in full three dimensions. However, modeling in 2D requires that waves are excited by sources whose distribution is infinitely expanded in the direction perpendicular to the cross-section. To accurately model a finite-source induced wave propagation, a 3D formulation is required [4]. Besides using Finite Differences (FDs) [5], conventional FEM [6] and its modifications (spectral FEM, semianalytical finite element method (SAFEM), the strip element method (SEM)), and the hybrid numerical method and modal expansion technique [7], several techniques dealing with the construction of the solution in the wavenumber-frequency domain and its inverse transformation into real space coordinates are considered. The analysis of wave propagation in three-dimensional structures requires at first the construction of the Green's matrix in the wavenumber-frequency domain and the investigation of dispersion properties. For the modeling of multilayered structures based on elasticity theory, the global matrix and transfer matrix methods [3,8,9], excitability matrices [4] or the Green's matrix of a multilayered plate can be used [10]. Inversion of expressions obtained in a 3D formulation in the transformed domain requires the computation of a double integral over wave numbers and a one-dimensional integral over frequencies. The time response is obtained through the frequency inversion of the Fourier transform using fast Fourier transform.

The most time-consuming procedure is the computation of the double integral over wave numbers, which causes difficulties such as integral singularity near real poles of Green's matrix, strong oscillations of the integrand and significant time expenses. Nevertheless, in the near- and middle-field of an excitation source, contour integrals of the inverse Fourier transform can be evaluated directly using some techniques for the computation of oscillatory integrals [11] or using adaptive two-dimensional numerical integration schemes [3,12]. But the computational costs are still considerable. Another disadvantage lies in the fact that the wave structure of the solution using these techniques is not taken into account and the wave field can not be analyzed for each Lamb wave mode separately.

Other methods for the computation of double wavenumber integral are based on the residue theorem, which allow to reduce

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computational costs considerably and to analyze the contribution of each Lamb wave mode. Wave propagation from surface-bonded piezoelectric actuators in an isotropic plate is considered in [13,8] using a residue theorem. A similar approach is originally offered for isotropic media in the case of an axisymmetric load [14]. When computing a wave propagation from surface-bonded piezo-actuators in an anisotropic layered plate, the authors in [8] substituted the integration along the positive real semi-axis by an integration along the whole real axis. The inverse transform is as before evaluated using the residue theorem. The double integral over wave numbers is reduced in [15] by the use of contour integration and the residue theorem to a one-dimensional integral, which is then computed numerically using standard quadratures. In [4] a far-field asymptotic expansion of related two-dimensional wavenumber integrals in terms of the modal solutions to the forced 2D problem is presented, whereas in [16] a far-field asymptotic expansion is obtained for the solution based on the Green's matrix of the composite plate [10]. Despite many publications referenced here, wave propagation from common types of sources of finite size anisotropic layered composites has not yet been completely analyzed. On of the main problems in these works is not accurate far-field solution of the problem near to caustics, where asymptotic expansions [4,16] doesn't work, moreover, in both works [4,16] no accurate solution in a middle field is provided.

In this paper harmonic wave propagation in a layered anisotropic plate with arbitrary anisotropy and arbitrary spatial orientation of each layer is studied. In the wavenumber-frequency domain, the solution of the problem is expressed in terms of Fourier transforms of the Green's matrix [10] and the surface load vector. For the computation of the inverse Fourier transform two techniques are used. The first one represents the displacements in terms of propagating modes, where all modes (propagating and evanescent) are taken into account. This technique imposes some restrictions on the type of loads; nevertheless, some practically important types of surface sources can be considered using this technique, e.g. the cases of circular and rectangular surface piezo actuators in [8,13]. The presented procedure for the computation of the wavenumber integral is in part similar to the one described in [15]. A significant difference lies in the fact that real singularities of the Fourier transform of the Green's matrix in [15] are analyzed in Cartesian coordinates, while in the present paper they are considered in polar coordinates. The second technique consists in applying the method of stationary phase to expressions obtained with the previous technique. It yields a far-field asymptotic solution for waves excited by surface sources of finite size similar to those obtained for point sources in [4]. The results obtained are validated by comparisons with results computed using a polynomial interpolation of the Green's matrix along the contour in the complex plane and an explicit evaluation of the contour integral [11]. Also a well coincidence of results obtained applying suggested in this paper techniques based on a residue theorem with experimental results from [17] is observed. The properties of Lamb wave propagation important for structural health monitoring (SHM) are discussed using numerical examples. The differences of the provided representations to other known solutions are also discussed. Another result of this paper consist in the accurate Airy function-based representation of the far-field solution of the wave propagation problem near to caustic directions.

2. Problem statement

In this paper the harmonic motion of a layered semi-infinite composite plate $-\infty \leqslant x_1,\ x_2 \leqslant \infty,\ x_3^{(n+1)} \leqslant x_3 \leqslant x_3^{(n)},\quad n = \overline{1,N}$, $x_3^{(1)} = 0,\ x_3^{(N+1)} = -h$ is considered. The value of h is the total thickness of the laminate (Fig. 1a). The wave propagation in each layer is given [18] by the following equations in global coordinates x_1, x_2, x_3 in the frequency domain (the common factor exp $(i\omega t)$ is omitted everywhere):

$$\frac{\partial \sigma_{jk}^{(n)}}{\partial x_k} + \rho^{(n)}\omega^2 u_j^{(n)} = 0, \tag{1}$$

$$\sigma_{ij}^{(n)} = C_{ijkm}^{(n)} \frac{\partial u_m^{(n)}}{\partial x_k}, \tag{2}$$

where i, j, k, m = 1, 2, 3, $n = \overline{1,N}$, $\sigma_{ij}^{(n)}$ are the coordinates of the stress tensor, $u_j^{(n)}$ are the components of the displacement vector, $\rho^{(n)}$ is the material density, $C_{ijkm}^{(n)}$ are the coordinates of the stiffness tensor, N is the number of layers, ω is the angular frequency. It should be noted that the layers can have different thicknesses. The forms of global spatial coordinates $x_1 = x$, $x_2 = y$, $x_3 = z$ are assumed to be equivalent. Here and elsewhere, unless otherwise specified, the rule of summation over repeated indices is used, and the upper index n denotes the number of the layer.

The waves in a structure are excited in a bounded domain Ω of the plane $z = 0$ (see Fig. 1a), where the surface stresses are assumed to be known

$$\sigma_{j3}^{(1)} = q_j(x,y)\exp(-i\omega t), \quad (x,y) \in \Omega, \quad (j = 1,2,3). \tag{3}$$

The lower boundary $z = z^{(N+1)} = -h$ of the layered composite is traction-free

$$\sigma_{13}^{(N)} = \sigma_{23}^{(N)} = \sigma_{33}^{(N)} = 0. \tag{4}$$

After the Fourier transform with respect to the coordinates x, y is applied, the solution of the boundary value problem (1)–(4) can be represented in the frequency-wavenumber domain $(\omega, \alpha_1, \alpha_2)$ as

$$U_i^{(n)}(\omega,\alpha_1,\alpha_2,z) = K_{ij}^{(n)}(\omega,\alpha_1,\alpha_2,z)Q_j(\alpha_1,\alpha_2), \quad i,\ j = 1,\ 2,\ 3. \tag{5}$$

Here $K_{ij}^{(n)}(\omega,\alpha_1,\alpha_2,z) = \mathcal{F}_{xy}\left[k_{ij}^{(n)}(\omega,x,y,z)\right]$ is the Green's matrix of the nth layer in the multilayered structure in the wavenumber domain. The vector $\mathbf{Q}$ with components $Q_j(\alpha_1,\alpha_2) = \mathcal{F}_{xy}[q_j(x,y)]$ is the load vector $\mathbf{q}(x,y)$ in the wavenumber domain. An algorithm to evaluate the Green's matrix in the frequency-wavenumber domain is described in detail in [10]. Similar procedures are also given in [3,8] using a transfer matrix method. Hereafter the parameters ω, z and n are assumed to be fixed, and the dependence of the Green's matrix $K_{ij}(\alpha_1,\alpha_2) \equiv K_{ij}^{(n)}(\omega,\alpha_1,\alpha_2,z)$ and the displacements $U_i(\alpha_1,\alpha_2) \equiv U_i^{(n)}(\omega,\alpha_1,\alpha_2,z)$ on these parameters is assumed to be implicit. Without loss of generality, the expressions below are valid for all values of these parameters (except for the static case $\omega = 0$).

To obtain the displacement vector $\mathbf{u}$, it is necessary to apply the inverse Fourier transform to the displacements $\mathbf{U}$ computed in the wavenumber domain:

$$\mathbf{u}(x,y) = \frac{1}{4\pi^2}\int_{\Gamma_1}\int_{\Gamma_2}\mathbf{K}(\alpha_1,\alpha_2)\mathbf{Q}(\alpha_1,\alpha_2)e^{-i(\alpha_1 x+\alpha_2 y)}d\alpha_1 d\alpha_2, \tag{6}$$

or in polar coordinates assuming $x = r\cos\varphi$, $y = r\sin\varphi$, $r = \sqrt{x^2+y^2}$ and $\alpha_1 = \alpha\cos\gamma,\ \alpha_2 = \alpha\sin\gamma,\ \alpha = \sqrt{\alpha_1^2+\alpha_2^2}$ as

$$\mathbf{u}(r,\varphi) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_{\Gamma^+(\gamma)}\mathbf{K}(\alpha,\gamma)\mathbf{Q}(\alpha,\gamma)e^{-i\alpha r\cos(\gamma-\varphi)}\alpha d\alpha d\gamma. \tag{7}$$

In this case, we assume that γ is real, $\gamma \in [0,2\pi]$, and α can also possess complex values. In formulas (6, 7), the symbols Γ_1, Γ_2 and $\Gamma^+(\gamma)$ denote the integration contours, which partially deviate from the real axis while bypassing the real poles of the Green's functions K_{ij} in accordance with the principle of limiting absorption [4,14]. This principle ensures physical meaning and uniqueness of the given solution. According to this principle, positive real poles of