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● *Original Contribution*

EXPERIMENTAL OBSERVATION OF CUMULATIVE SECOND-HARMONIC GENERATION OF LAMB WAVES PROPAGATING IN LONG BONES

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Abstract—The experimental observation of cumulative second-harmonic generation of fundamental Lamb waves in long bones is reported. Based on the modal expansion approach to waveguide excitation and the dispersion characteristics of Lamb waves in long bones, the mechanism underlying the generation and accumulation of second harmonics by propagation of the fundamental Lamb waves was investigated. An experimental setup was established to detect the second-harmonic signals of Lamb wave propagation in long bones *in vitro*. Through analysis of the group velocities of the received signals, the appropriate fundamental Lamb wave modes and the duration of the second-harmonic signals could be identified. The integrated amplitude of the time-domain second-harmonic signal was introduced and used to characterize the efficiency of second-harmonic generation by fundamental Lamb wave propagation. The results indicate that the second-harmonic signal generated by fundamental Lamb waves propagating in long bones can be observed clearly, and the effect was cumulative with propagation distance when the fundamental Lamb wave mode and the double-frequency Lamb wave mode had the same phase velocities. The present results may be important in the development of a new method to evaluate the status of long bones using the cumulative second harmonic of ultrasonic Lamb waves. (E-mail: tda@fudan.edu.cn or tadean.fudan@gmail.com and dengmx65@yahoo.com) © 2014 World Federation for Ultrasound in Medicine & Biology.

Key Words: **Lamb waves, Dispersion, Second-harmonic generation, Long bones.**

INTRODUCTION

The ultrasonic axial transmission technique has attracted considerable interest for the assessment of long bones (Grondin et al. 2012; Le et al. 2010; Muller et al. 2005a; Naili et al. 2010; Raum et al. 2005). In this technique, ultrasonic transmitters and receivers are placed on the same side of long bones. Restricted by the strong boundary conditions among bone layers, soft tissue and marrow, ultrasonic waves propagating in the axial direction are thus able to reflect all of the properties of long bones along the propagation distance. The so-called first-arriving signal has traditionally been used to determine several parameters of long bones, such as mineral density and bone geometry (Chen et al. 2013; Grondin et al. 2010; Haïat et al. 2009; Machado et al. 2010, 2011; Zheng et al. 2007). In addition to the first-arriving signal, long bones also support the propagation of ultrasonic guided waves, as they are approximately cylindrical structures. It has been found that ultrasonic guided waves propagate throughout the thickness of long bones, providing more comprehensive information on bone material and structural characteristics (Ta et al. 2009). The mechanism underlying the propagation of guided waves in long bones has been clearly delineated using theoretical analyses, simulations and experimental methods (Bossy et al. 2004a, 2004b; Gheduzzi et al. 2009; Kilappa et al. 2013; Moilanen et al. 2006; Ta et al. 2006; Talmant et al. 2009). Several efficient algorithms have also been proposed to analyze or separate multi-modal guided-wave signals in long bones (Minonzio et al. 2010; Song et al. 2011, 2012; Zhang and Ta 2012a, 2012b; Zhang et al. 2013; Xu et al. 2010, 2012). Furthermore, several researchers have applied ultrasonic guided waves to evaluate the status of long bones, with some significant results (Baron 2012; Bossy et al. 2004a, 2004b; Grimal et al. 2013; Kilappa et al. 2011; Moilanen et al. 2007; Tatarinov et al. 2011; Zhang and Ta 2012a, 2012b).

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Almost all of the above-mentioned studies, however, have been limited to the linear characteristics of ultrasonic wave propagation in long bones. It is known that the material's elastic non-linearity is, in general, more sensitive to some defects than its elastic linearity (Bermes et al. 2007; Deng 2006; Deng et al. 2011; Müller et al. 2010). Recently, some non-linear elastic wave techniques have been used to assess bone damage status, including non-linear resonant ultrasound spectroscopy (Muller et al. 2005b, 2008), non-linear dynamical responses (Ulrich et al. 2007), time-of-flight modulation measurements (Renaud et al. 2008a), parametric generation (Renaud et al. 2008b), non-linear ultrasound vibro-modulation (Zacharias et al. 2009) and dynamic acousto-elastic testing (Moreschi et al. 2011). Moreover, investigations pertaining to the process of second-harmonic generation by fundamental Lamb wave propagation in planar structures have been reported (Deng 1999, 2003, 2006; Deng and Pei 2007; Lima and Hamilton 2003). It has been found that under the conditions that the transfer of energy from the fundamental Lamb wave to the double-frequency Lamb wave (DFLW) is not zero and that the phase velocity matching is satisfied, the second-harmonic signals of Lamb wave propagation can be observed clearly. The non-linear Lamb wave technique combines the high sensitivity of non-linear ultrasonic measurements and the expeditiousness of Lamb wave inspection, and it has been found to have great potential for the assessment of planar structures (Bermes et al. 2007; Deng 2006; Deng and Pei 2007).

The objective of the present work was to observe experimentally the second-harmonic signals of the fundamental Lamb waves propagating in long bones. First, phase velocity matching between the fundamental Lamb wave and the DFLW was introduced to guarantee the cumulative second-harmonic generation. Second, an experimental setup was established to generate and measure the second-harmonic signals in both an aluminum plate and four specimens of bovine tibia. Finally, the group velocity curves of the fundamental Lamb waves and the DFLWs were calculated to enable an analysis of the experimental results. The cumulative growth effect of second harmonics was experimentally verified according to the quantitative relationship between the integrated amplitude of the second harmonic and the propagation distance.

THEORETICAL FUNDAMENTALS

As illustrated in Figure 1, when a fundamental Lamb wave (with driving frequency *f*) travels down a stress-free solid plate (with thickness th) along the *z*-axis, within the second-order perturbation, there is a bulk driving force of double the fundamental frequency in the solid plate [denoted by $\boldsymbol{F}_{\mathrm{B}}^{(2f)}$] because of the convective non-linearity

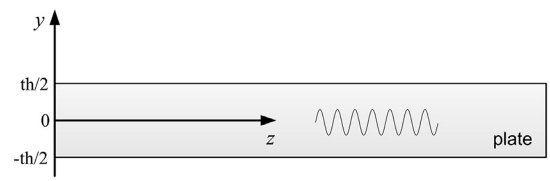

Fig. 1. Lamb wave propagation in a stress-free solid plate of thickness th.

(because of the description difference between Lagrangian and Eulerian coordinates) and the inherent non-linear elastic properties of the solid (Deng et al. 2011). For simplicity, the anisotropy and attenuation of the plate material are not considered. The *y*- and *z*-axes are assumed to be normal and parallel to the surface of the solid plate, respectively. The form of $\boldsymbol{F}_{\mathrm{B}}^{(2f)}$ associated with the fundamental Lamb wave is given in the Appendix (Deng 1999). Besides the existence of $\boldsymbol{F}_{\mathrm{B}}^{(2f)}$ in the interior of the solid plate, there are traction stress tensors of double the fundamental frequency at the two surfaces of the solid plate, denoted by $\boldsymbol{P}_{\mathrm{S}}^{(2f)}$; its components (denoted by $P_{yy}^{(2f)}$ and $P_{zy}^{(2f)}$) are listed in the Appendix (Deng 2003; Lima and Hamilton 2003). Both $\boldsymbol{F}_{\mathrm{B}}^{(2f)}$ and $\boldsymbol{P}_{\mathrm{S}}^{(2f)}$ are associated with the mass density and second- and third-order elastic constants of the plate material, and are proportional to the square of the amplitude of the fundamental Lamb wave.

According to the modal expansion analysis approach to waveguide excitation, the second-harmonic field of fundamental Lamb wave propagation can be regarded as the linear superposition of a series of DFLW components generated by both $\boldsymbol{F}_{\mathrm{B}}^{(2f)}$ and $\boldsymbol{P}_{\mathrm{S}}^{(2f)}$. It has been found that only symmetric DFLW modes can be generated, regardless of whether the fundamental Lamb wave mode is symmetric or anti-symmetric (Deng 1999, 2003). For a specific fundamental Lamb wave mode, its second-harmonic field, denoted by $\boldsymbol{U}^{(2f)}$, can be expressed as (Deng 2003; Lima and Hamilton 2003)

$$\boldsymbol{U}^{(2f)} = \sum_{n} A_n(z) \times \boldsymbol{U}_n^{(2f)}(y) \tag{1}$$

where $\boldsymbol{U}_n^{(2f)}(y)$ is the function of the mechanical displacement of the *n*th DFLW mode component, and $A_n(z)$ is the corresponding expansion coefficient. When the excitation source generating the fundamental Lamb wave mode is assumed to be located at $z = 0$, the magnitude of $A_n(z)$ can formally be found to be (Deng et al. 2011)

$$|A_n(z)| = \left| \frac{f_{Vn}^{(2f)}(z) + f_{Sn}^{(2f)}(z)}{4P_{nn}} \times \frac{c_{\mathrm{p}}^{(f)} c_{\mathrm{p}}^{(2f)}}{2\pi f \left[c_{\mathrm{p}}^{(2f)} - c_{\mathrm{p}}^{(f)} \right]} \times \sin\left\{ \frac{2\pi f \left[c_{\mathrm{p}}^{(2f)} - c_{\mathrm{p}}^{(f)} \right]}{c_{\mathrm{p}}^{(f)} c_{\mathrm{p}}^{(2f)}} \times z \right\} \right| \tag{2}$$