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On the stability of triangular points in the relativistic R3BP with oblate primaries and bigger radiating

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Abstract

We consider a version of the relativistic R3BP which includes the effects of oblateness of the primaries and radiation of the bigger primary as well on the stability of triangular points. We observe that the positions of the triangular points and their stability are affected by the relativistic effect apart from the radiation and oblateness of the primaries. It is further seen for these points that the range of stability region increases or decreases according as the part of the critical mass value, depending upon relativistic terms, radiation and oblateness coefficients, is positive or negative. A numerical exploration shows that in the Sun–Saturn, Sun–Uranus, Sun–Neptune systems, the oblateness has no influence on their positions and range of stability region; whereas it has a little influence on the Sun–Mars, Sun–Jupiter systems. On the other hand, we found that radiation pressure has an observable effect on the solar system.

Keywords: Celestial mechanics; Oblateness; Radiation; Relativity; R3BP

1. Introduction

The circular restricted three-body problem (CR3BP) is a famous classical problem and has been receiving considerable attention of scientists and astronomers because of its applications in the dynamics of the solar and stellar systems, lunar theory and artificial satellites. It possesses three collinear $L_{1,2,3}$ and two triangular $L_{4,5}$ points of equilibrium. The former points are in general unstable, while the latter points are stable for the mass ratio $\mu < 0.03852\ldots$ (Szebehely, 1967).

Over the years this problem has been unable to discuss the motion of a test particle when its bodies concerned are luminous or oblate spheroids or of variable mass. In the last several decades, there has been strong revival of interest in the restricted problem. To make the problem more realistic several perturbing agents have been included in the study of the R3BP. Radiation and oblateness have been main subjects of various investigations (Radzievskii, 1950; Schuerman, 1972, 1980; SubbaRao and Sharma, 1975; Sharma and SubbaRao, 1975; Bhatnagar and Hallan, 1979; Kunitsyn and Polyakhova, 1995; Oberti and Vienne, 2003; AbdulRaheem and Singh, 2006; Singh, 2011; Abouelmagd, 2013).

The relativistic effect also plays a key role in the CR3BP. Brumberg (1972, 1991) studied the relativistic problem of three bodies in more details and collected most of the important results on relativistic celestial mechanics. He not only obtained the equations of the motion for the general problem of three bodies but also deduced the equation of motion for the restricted problem of three bodies.

Maindl and Dvorak (1994) derived the equations of motion for the relativistic R3BP using the post-Newtonian approximation of relativity. Their equations

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depend on the mass parameter μ and $\frac{M}{R}$, where M is the mass of the system and R is the distance between the primaries. This parameter $\frac{M}{R}$ is thus a kind of measure for the role of relativistic effects in the system (the classical case is $\frac{M}{R}=0$). Finally, they applied the model to the computation of the advance of Mercury's perihelion in solar system and leads to results compatible with published data.

Bhatnagar and Hallan (1998) investigated the existence and linear stability of the triangular points $L_{4,5}$ in the relativistic R3BP, and found that $L_{4,5}$ are always unstable in the whole range $0 \leqslant \mu \leqslant \frac{1}{2}$, in contrast to the classical R3BP in which they are stable for $\mu < \mu_0$, where μ is the mass ratio and $\mu_0 = 0.038520\ldots$ is the Routh's value.

Douskos and Perdios (2002) studied the stability of the triangular points in the relativistic R3BP and contrary to the result of Bhatnagar and Hallan (1998), they obtained a region of linear stability in the parameter space as $0 \leqslant \mu < \mu_0 - \frac{17\sqrt{69}}{486c^2}$ where μ_0 is Routh's value. They also determined the positions of the collinear points and showed that they are always unstable.

Abd El-Bar and Abd El-Salam (2012) studied the effects of relativistic R3BP on both triangular and collinear equilibrium points. The approximate locations of the collinear and triangular points are determined. Recently, many perturbing forces, such as radiation, oblateness, centrifugal force have been included in the study of the relativistic R3BP.

Abd El-Salam and Abd El-Bar (2014) studied the photogravitational restricted three-body problem within the framework of the post-Newtonian approximation. The mass of the primaries are assumed to change under the effect of continuous radiation process. The locations of the triangular points are computed. Series forms of the locations are obtained as new analytic results.

Katour et al. (2014) extended this work by including the effect of oblateness of both primaries. They computed the new perturbed locations of the triangular points. Singh and Bello (2014a) investigated the motion of a test particle in the vicinity of the triangular points $L_{4,5}$ by considering the more massive primary as a source of radiation in the framework of the relativistic restricted three-body problem (R3BP). They found that the position and stability of the triangular point are affected by both the relativistic factor and radiation pressure. In their further paper (Singh and Bello, 2014b) they studied the motion of a test particle (infinitesimal mass) in the neighborhood of the triangular point L_4 in the framework of the perturbed relativistic restricted three-body (R3BP). The problem is perturbed in the sense that a small perturbation is given to the centrifugal force. They found that the position and stability of the triangular point are affected by both the relativistic factor and a small perturbation in the centrifugal force.

To the present authors' Knowledge no investigation has been carried on the stability of equilibrium points in the relativistic R3BP with oblate primaries and the bigger one radiating. Hence, we thought to study the stability of triangular points of this problem. This model is more realistic for our solar system because the Sun is a source of radiation as well as oblate and the most of its planets are oblate.

This paper proceeds as follows: in Section 2, the equations governing the motion are presented; Section 3 describes the positions of triangular points, while their linear stability is analyzed in Section 4; the discussion is given in Section 5, the numerical applications are given in Section 6, finally Section 7 conveys the main findings of this paper.

2. Equations of motion

The pertinent equations of motion of an infinitesimal mass in the relativistic R3BP when the primaries are oblate spheroids as well as the bigger primary is a source of radiation, in a barycentric synodic coordinate system (ξ, η) and dimensionless variables, can be written as Brumberg (1972) and Bhatnagar and Hallan (1998):

$$\begin{aligned}
\ddot{\xi} - 2n\dot{\eta} &= \frac{\partial W}{\partial \xi} - \frac{d}{dt}\left(\frac{\partial W}{\partial \dot{\xi}}\right) \\
\ddot{\eta} + 2n\dot{\xi} &= \frac{\partial W}{\partial \eta} - \frac{d}{dt}\left(\frac{\partial W}{\partial \dot{\eta}}\right)
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
W = {} & \frac{1}{2}n^2(\xi^2+\eta^2) + \frac{q_1(1-\mu)}{\rho_1}\left(1+\frac{A_1}{2\rho_1^2}\right) + \frac{\mu}{\rho_2}\left(1+\frac{A_2}{2\rho_2^2}\right) \\
& + \frac{1}{c^2}\left[\frac{1}{8}\left\{\dot{\xi}^2+\dot{\eta}^2+2n(\xi\dot{\eta}-\eta\dot{\xi})+n^2(\xi^2+\eta^2)\right\}^2\right. \\
& + \frac{3}{2}\left(\frac{q_1(1-\mu)}{\rho_1}\left(1+\frac{A_1}{2\rho_1^2}\right)+\frac{\mu}{\rho_2}\left(1+\frac{A_2}{2\rho_2^2}\right)\right) \\
& \left\{\dot{\xi}^2+\dot{\eta}^2+2n(\xi\dot{\eta}-\eta\dot{\xi})+n^2(\xi^2+\eta^2)\right\} \\
& - \frac{1}{2}\left(\frac{q_1^2(1-\mu)^2}{\rho_1^2}\left(1+\frac{A_1}{2\rho_1^2}\right)^2+\frac{\mu^2}{\rho_2^2}\left(1+\frac{A_2}{2\rho_2^2}\right)^2\right) \\
& + q_1\mu(1-\mu)\left\{n\left(4\dot{\eta}+\frac{7}{2}n\xi\right)\left(\frac{1}{\rho_1}\left(1+\frac{A_1}{2\rho_1^2}\right)\right.\right. \\
& \left. - \frac{1}{\rho_2}\left(1+\frac{A_2}{2\rho_2^2}\right)\right) - \frac{n^2}{2}\eta^2\left(\frac{\mu}{\rho_1^3}+\frac{q_1(1-\mu)}{\rho_2^3}\right) \\
& + n^2\left(-\frac{1}{\rho_1\rho_2}+\frac{\mu}{2\rho_1}+\frac{q_1(1-\mu)}{2\rho_2}-\frac{q_1(1-\mu)}{\rho_1}\left(1+\frac{A_1}{2\rho_1^2}\right)\right. \\
& \left.\left.\left. - \frac{\mu}{\rho_2}\left(1+\frac{A_2}{2\rho_2^2}\right)\right)\right\}\right]
\end{aligned} \tag{2}$$