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Transfer orbits to the Earth–Moon triangular libration points

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Abstract

The particular positions and dynamics of the triangular libration points in the Earth–Moon system make them potential candidates for future space applications. Taking the leading L_4 point as an example, this paper studies the transfer orbits to the vicinity of this equilibrium point. Two basic models are used: the circular restricted three-body problem (CRTBP) and the bi-circular problem (BCP). The order-three analytical solution of the motion around the triangular libration points in the CRTBP model is taken as the nominal orbit. Three different approaches are studied: direct transfer, transfer utilizing powered lunar gravity assist, and transfer utilizing the Sun's gravity. Lastly, low energy transfer orbits are extensively studied in the BCP model via a numerical approach. Our studies show that the total delta-v cost is considerably reduced if the Moon's gravity or the Sun's gravity can be used, at the cost of a longer transfer time. The delta-v cost and the time of flight (TOF) in our work are approximately: 3.90–4.40 km/s and 4–8 days for the direct transfer; 3.41–3.48 km/s and 18–52 days for the transfer utilizing powered lunar gravity assist; 3.33–3.40 km/s and 70–95 days for the transfer utilizing the Sun's gravity. For the low energy transfer orbits, the lower limit of the delta-v cost is around 3.10 km/s.

Keywords: Triangular libration point; Earth–Moon; Gravity assist; Invariant manifold

1. Introduction

The collinear libration points of the CRTBP model are popular candidates for some space applications. Currently, these applications are still restricted to the Sun–Earth system and the Earth–Moon system (Farquhar et al., 2004). Probes can stay nearby these points with orbit control for quite a long time to perform some astronomical observations or other applications. Besides playing the role of a platform for probes, the stable and unstable dynamical structures (known as invariant manifolds) of these points are critical to low-energy interplanetary transfers (Koon et al., 2000; Gómez et al., 2001b).

The triangular libration points are another type of equilibrium point in the CRTBP. Different from their collinear siblings, these points are linearly stable when the mass parameter is less than the Routh critical value ($\mu_c = 0.03852\cdots$, see Szebehely (1967)), and are nonlinearly stable (planar case) except for few critical values (Deprit and Deprit-Bartholomé, 1967). These points are extensively studied in astronomy because they are the locations of a unique type of small bodies in the solar system—the Trojans (Yoshida and Nakamura, 2008; Hou et al., 2014a, b). However, except a few works (Catlin and McLaughlin, 2007), these points are seldom studied in space applications. Nevertheless, their special locations along with their relatively better stability property indicate their potential applications in future space missions. For example, the very long distance between a triangular libration point probe and a ground station, or between two triangular libration point probes, can form the space VLBI (very long baseline interferometry) and can greatly enhance the accuracy of the observations (Hirabayashi and Hirosawa, 2000).

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In the CRTBP model of the Earth–Moon system, the triangular points are marginally stable. But in the real force model where the perturbations from the Moon's orbit eccentricity and solar gravity cannot be neglected, these points are unstable (Gómez et al., 2001a; Hou and Liu, 2010). The instability is much milder when compared with the collinear libration points (Hou and Liu, 2011). As a result, a much lower station keeping frequency should be expected. While the better stability property is an advantage compared with the collinear points, it's also a disadvantage that there are no associated stable or unstable invariant manifolds which can be used as passages to and from these points. But this is not an inherent difficulty, because we can still reach the proximity of these points even through the direct transfer orbit (similar to the Hohmann transfer orbit in the two body problem; see Section 3 for details). Nevertheless, in designing transfer orbits to these points, lunar gravity assist can be used to reduce the energy cost (Broucke, 1979; Prado, 1996; Gómez et al., 2001a; Belbruno and Miller, 1993; Salazar et al., 2012, 2014; Lei and Xu, 2014).

In this paper, we carryout a systematic study on the transfer orbits to these libration points. Firstly, The direct transfer orbits to the triangular libration point orbits are studied and classified into four types. The problem was qualitatively addressed in previous works (Prado, 1996), with an estimation of the delta-v cost but no details. The current work extends the study. The conclusion that one type of direct transfer is better than the other three types can be used as guidance for future space missions to these points.

Secondly, while most lunar gravity assist strategies are proposed by the patched conic method based on the two-body problem (Prado and Broucke, 1995; Salazar et al., 2014), we propose a new strategy to design the powered lunar gravity assisted transfer orbit. It is based on the CRTBP model of the Earth–Moon system. The orbit (mainly the part departing from the perilune to the triangular libration point) is not restrained by the two-body relation. The delta-v cost can be significantly reduced compared with the orbits in previous works (Salazar et al., 2014). In our strategy, usually a small maneuver at the perilune is needed, so we call this kind of transfer as powered lunar gravity assist (PLGA) transfer. Then, ignoring lunar gravity, the transfer orbit under the perturbation of the Sun is studied to find out how much delta-v can be saved by utilizing the Sun's gravity. This approach is different from previous studies which simultaneously consider the Moon's gravity and the Sun's gravity. With this approach, we're able to evaluate the contribution from the Sun and compare it with PLGA. Studies show that the insertion maneuver into the nominal orbit is a little bit larger than that of the PLGA transfer. As for the maneuver departing from the parking low-Earth orbit (LEO), there is no obvious difference. But since there is another small maneuver for the PLGA transfer orbit at the perilune in our study, the total delta-v cost of the PLGA transfer and the transfer utilizing the Sun's gravity are comparable to each other. As for the TOF, since the PLGA transfer orbits are generally confined in the Earth–Moon system, usually they need less time than the transfer utilizing the Sun's gravity.

Lastly, by using the BCP model, low energy transfer orbits are extensively studied via a numerical approach. These low energy transfer orbits utilize the Moon's gravity, the Sun's gravity or both. The TOF for these orbits is generally very long. Some example trajectories with extreme low delta-v cost are presented. A plot of the delta-v cost versus the TOF is given. The lower limit of the delta-v cost from our numerical results is around 3.10 km/s. This value is compared with analytical results.

2. Dynamical models

Two force models are considered in our study: the CRTBP model and the BCP model. In this section, a brief introduction to the two models will be given. Without specifications, following units are used in our studies: the mass unit is the sum of the masses of the Earth and the Moon. The length unit is the mean distance between the Earth and the Moon (384,400 km). The time unit is chosen such that the gravitational constant $G = 1$ (4.3483 days). The integrator used is the RKF78 one, with a truncation tolerance of 10^{-14}.

2.1. The CRTBP model

To simplify the studies, only planar trajectories are considered throughout our work, as did by many previous works (Broucke, 1979; Prado, 1996; Salazar et al., 2012, 2014; Lei and Xu, 2014). Study in the three-dimensional space can provide more insights into the dynamics of the transfer orbits, but will also increase the difficulty due to the extra out-of-plane freedom, and in the authors' opinion will not qualitatively change the conclusions in this paper.

In this model, Earth and Moon (or Sun and Earth) are the two primaries and the probe is the small body. In the barycentric synodic frame and using the units described above, the equations of motion (EOM) of the planar CRTBP model are (Szebehely, 1967):

$$\ddot{x} - 2\dot{y} = \partial\Omega/\partial x, \ddot{y} + 2\dot{x} = \partial\Omega/\partial y, \quad (1)$$

where x, y are coordinates of the small body in the barycentric synodic frame, and

$$\Omega = (x^2 + y^2)/2 + (1-\mu)/r_1 + \mu/r_2.$$

The two variables r_1, r_2 are distances of the small body from the two primaries. The mass parameter is defined as $\mu = m_2/(m_1 + m_2)$, where m_1, m_2 are masses of the two primaries. There is an integral of the motion (the Jacobi's integral), with the form of:

$$C = 2\Omega - v^2 = 2\Omega - (\dot{x}^2 + \dot{y}^2), \quad (2)$$

where C is the Jacobi constant.