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The effect of zonal harmonic coefficients in the framework of the restricted three-body problem

Elbaz I. Abouelmagd [a,b,c,*], M.S. Alhothuali [c], Juan L.G. Guirao [d], H.M. Malaikah [c]

[a] Mathematics Department, Faculty of Science and Arts (Khulais), University of Jeddah, Saudi Arabia
[b] National Research Institute of Astronomy and Geophysics, Cairo, Egypt
[c] Nonlinear Analysis and Applied Mathematics Research Group (NAAM), Mathematics Department, King Abdulaziz University, Jeddah, Saudi Arabia
[d] Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena, Hospital de Marina, 30203 Cartagena, Región de Murcia, Spain

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Abstract

The objective of this paper is to present a comprehensive analytical study on the existence of the libration points and their linear stability in the frame of the restricted three-body problem considering the effect of the first two even zonal harmonics parameters with respect to both primaries. Moreover, the periodic orbits around the libration points, the expressions for semi-major and semi-minor axes, the eccentricities and the periods of elliptical orbits as well as the orientation of the principal axes are stated. In addition, we support our study with some numerical and graphical experiments.

Keywords: Restricted three-body problem; Libration points; Stability; Zonal harmonics coefficients; Periodic orbits

1. Introduction

The three-body problem is one of the central problems in field of Celestial Mechanics. It has many applications in different scientific areas, in particular, in the fields of Astrophysics and astrodynamics. This problem is classified in two classes: the first one is the general problem which describes the motion of three celestial bodies under their mutual gravitational attraction. The second class is the *restricted problem* in which the third body has an infinitesimal mass compared with masses of the other two bodies and consequently it does not affect their motion.

One of the main applications of the general problem in Astrophysics is for instance the dynamics of triple stars systems. In the second half of the 20th century and even today. the study of scientific community has center its attention on the restricted three-body problem and there are a big number of papers studying different aspect of this problem. For instance, considering the influences of perturbed forces such as oblateness, radiation pressure, Coriolis and centrifugal forces, variation of masses, the Pointing–Robertson effect, the atmospheric drag, the solar wind,. . ., etc.

Significant studies related with the libration points considering the oblateness of one or both primaries when the equatorial plane is coincident with the plane of motion are done by Subbarao and Sharma (1975), Sharma and Subbarao (1978) and Markellos et al. (1996).

Some works studying different aspects of the dynamics of the restricted problem when the three participating

* Corresponding author at: Mathematics Department, Faculty of Science and Arts (Khulais), University of Jeddah, Saudi Arabia.
E-mail addresses: eabouelmagd@gmail.com, eabouelmagd@kau.edu.sa (E.I. Abouelmagd), juan.garcia@upct.es (J.L.G. Guirao).
URL: http://www.jlguirao.es (J.L.G. Guirao).

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bodies are oblate spheroids are given by El-Shaboury and El-Tantawy (1993), Abouelmagd and El-Shaboury (2012) and Elipe and Ferrer (1985). Interesting papers when one or both primaries are triaxial bodies are for instance El-Shaboury et al. (1991), Khanna and Bhatnagar (1999) and Sharma and Bhatnagar (2001).

Several authors have been devoted their efforts to study the effects of small perturbations in centrifugal and Coriolis forces as Szebehely (1967), Bhatnagar et al. (1978), Devi and Singh (1994) and Shu et al. (2005).

The existence and the linear stability of the libration points in the restricted problem for perturbed potentials between the bodies in the cases: the bigger primary is an oblate spheroid, both primaries are oblate spheroids, the primaries are spherical and the biggest primary is a source of radiation were studied in by Bhatnagar and Hallan (1979). They observed that the collinear points are unstable, the range of stability for the triangular points increases or decreases depending on the sign of a parameter which depends on the perturbed functions. Furthermore, Singh and Ishwar (1999) studies the effect of the oblateness and the radiation pressure at the locations of the triangular points and their linear stability when both the primaries are oblate and radiating.

Ishwar and Elipe (2001) found the secular solutions at the triangular points in the generalized photogravitational restricted three-body problem. The problem is generalized in the sense that the bigger primary is a source of radiation and the smaller one is an oblate spheroid. Moreover, Abouelmagd et al. (2014a) found the secular solution around the triangular equilibrium points and reduce it to a periodic solution in the frame work of the generalized restricted three-body problem, in sense that both primaries are oblate and radiating as well as the gravitational potential from a belt. They also showed that the linearized equation of motion of the infinitesimal body around the triangular equilibrium points has a secular solution when the value of the masses ratio is equal to the critical mass value. Numerical and graphical analysis in order to understand the effects of the perturbed forces are stated.

Mittal and Bhatnagar (2009) studies the periodic orbits generated by Lagrangian solutions of the restricted three-body problem when the bigger body is a source of radiation and the smaller is an oblate spheroid. It is used the definition of Karimov and Sokolsky (1989) for mobile coordinates to determine these orbits and the predictor method to draw them.

Singh and Begha (2011) studied the existence of periodic orbits around the triangular points in the restricted three-body problem when the bigger primary is triaxial and the smaller one is considered as an oblate spheroid. In the range of linear stability under the effects of the perturbed forces of Coriolis and centrifugal, it is deduced that long and short periodic orbits exist around these points and are stated their periods, orientation and eccentricities affected by the non sphericity and the perturbations in the Coriolis and centrifugal forces.

Abouelmagd (2012) studies the effects of oblateness coefficients J_2 and J_4 of the bigger primary in the planar restricted three-body problem on the locations of the triangular points and their linear stability. It was found that these locations are affected by the coefficients of oblateness. Furthermore, it was stated that the triangular points are stable for $0 < \mu < \mu_c$ and unstable when $\mu_c \leqslant \mu \leqslant 1/2$, where μ_c is the critical mass parameter which depends on the coefficients of oblateness. Some numerical values for the positions of the triangular points for certain planets of the solar system were stated remarking that the range of stability decrease. Finally, some examples showing that there is no influence for J_4 on the range of stability for some planets systems as Earth–Moon, Saturn–Phoebe and Uranus–Caliban systems. Furthermore, the existence of new equilibrium points for the restricted three-body problem with equal prolate primaries is stated in Douskos et al. (2012). It was found that these points are located on the Z-axis above and below the inner Eulerian equilibrium point L_1 as well as a new type of straight-line periodic oscillations, different from the well-known Sitnikov motions. The stability properties of these oscillations are used to find new types of families of 3D periodic orbits branch out of the Z-axis consisting of orbits located entirely above or below the orbital plane of the primaries. Recently, there are also some interesting papers connected with the restricted three-body problem, see Beevi and Sharma (2012), Singh (2012), Singh and Taura (2013), Kishor and Kushvah (2013), Abouelmagd et al. (2013), Abouelmagd (2013) and Abouelmagd et al. (2014b).

In last decades a great number of authors have studied the restricted three-body problem taking account the effects of oblateness of one or both primaries up to 10^{-3} of the main terms of the potential. But our main contribution and major modification in the present work is to study the effect of oblateness up to 10^{-6} when both primaries are oblate spheroids. Therefore we shall consider the influence of even zonal harmonic parameters up to J_4 for both primaries on the existence of the libration points and their linear stability as well as we shall analyze the existence of periodic orbits around these points.

This work is organized as follow: a historical review on the importance of the three-body problem and the aim of the present work is presented in current Section. The mean motion and equations of motion of the problem under consideration are derived in Sections 2 and 3. In Section 4 we find the locations of the libration points and in Section 5 we present a study of their linear stability. In Section 6 is obtained the expression of critical mass and in Section 7 this notion is extended to find periodic orbits around libration points. Finally, some pictures showing the conclusions of our study are presented in the last section. We underline that the model studied in this work has special importance in space missions either to send telescopes or for dispatching satellites or exploring vehicles to stable regions to move in gravitational fields of planets systems.