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Advances in Space Research 56 (2015) 1264–1275

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

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Finite gyroradius corrections in the theory of perpendicular diffusion 1. Suppressed velocity diffusion

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Received 22 April 2015; received in revised form 4 June 2015; accepted 19 June 2015
Available online 25 June 2015

Abstract

A fundamental problem in plasma physics, space science, and astrophysics is the transport of energetic particles interacting with stochastic magnetic fields. In particular the motion of particles across a large scale magnetic field is difficult to describe analytically. However, progress has been achieved in the recent years due to the development of the unified non-linear transport theory which can be used to describe magnetic field line diffusion as well as perpendicular diffusion of energetic particles. The latter theory agrees very well with different independently performed test-particle simulations. However, the theory is still based on different approximations and assumptions. In the current article we extend the theory by taking into account the finite gyroradius of the particle motion and calculate corrections in different asymptotic limits. We consider different turbulence models as examples such as the slab model, noisy slab turbulence, and the two-dimensional model. Whereas there are no finite gyroradius corrections for slab turbulence, the perpendicular diffusion coefficient is reduced in the other two cases. The matter investigated in this article is also related to the parameter a^2 occurring in non-linear diffusion theories.

Keywords: Magnetic fields; Turbulence; Energetic particles

1. Introduction

Energetic particles interact with turbulent magnetic fields while they propagate through a magnetized plasma. Due to this interaction, their motion is a stochastic motion. In addition to turbulent fields $\delta\vec{B}$, there are also large scale magnetic fields $\vec{B}_0$ influencing the particle orbit. The latter field is usually called guide field, mean field, background field, or large scale field. This type of configuration can be found in different physical environments such as fusion devices, the solar wind, or the interstellar medium (see, e.g., Schlickeiser (2002), Spatschek (2008), and Shalchi (2009) for reviews). The turbulent fields described above lead to a diffusive motion of the particles. Due to the large scale field one has to distinguish between diffusion along and across that field. It is often assumed that perpendicular diffusion is more difficult to describe analytically compared to parallel transport. It should be noted that sub- and superdiffusive transport has been discussed more recently in the literature (see, e.g., Zimbardo et al. (2006), Pommois et al. (2007), Shalchi and Kourakis (2007), and Zimbardo et al. (2012)).

Previous theories for perpendicular diffusion such as the Non-Linear Guiding Center (NLGC) theory of Matthaeus et al. (2003) and the Unified Non-Linear Transport (UNLT) theory of Shalchi (2010) neglect the rotation of the particle in the direction perpendicular with respect to the large scale field. Furthermore, as equation of motion in such theories, the following *Ansatz* is used

$$V_x = a v_z \delta B_x[\vec{x}(t), t]/B_0. \quad (1)$$

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http://dx.doi.org/10.1016/j.asr.2015.06.018

Here we have used the particle position $\vec{x}(t)$ at time t, the z-component of the particle velocity vector v_z, the x-component of the turbulent magnetic field δB_x, the mean magnetic field B_0, and the x-component of the guiding center velocity vector V_x (see Section 2 for more details). The parameter "a" used here can be seen as an unknown parameter. If non-linear theories are compared with test-particle simulations, best agreement is usually found for values between $a^2 = 1/3$ and $a^2 = 1$ (see, e.g., Matthaeus et al. (2003) and Tautz and Shalchi (2011)). However, analytical work based on the Newton–Lorentz equation suggests that this parameter is between $a^2 = 1$ and $a^2 = 2$ (see Shalchi and Dosch (2008), Dosch and Shalchi (2009), and Dosch et al. (2013)).

Especially the UNLT theory has shown remarkable agreement with test-particle simulations for different turbulence models such as the slab/2D model, the Goldreich-Sridhar model, Alfvén waves, and noisy reduced MHD turbulence (see Tautz and Shalchi (2011), Shalchi (2013), Hussein and Shalchi (2014), Shalchi and Hussein (2014), and Shalchi and Hussein (2015)). Therefore, we conclude that the main problem in the theory of perpendicular transport is Eq. (1) and therewith the parameter "a".

In the current paper we explore the influence of finite gyroradius effects. To do this we employ two different approaches, namely:

(1) Quasi-Linear Theory (QLT, see Jokipii (1966)),
(2) Non-linear diffusion theory (see, e.g., Shalchi et al. (2004) and Shalchi (2010));

For both approaches we compute the perpendicular diffusion coefficient as a function of the gyroradius. Results are compared with each other and previous findings are recovered by considering appropriate limits. Furthermore, we estimate the value of "a" in the context of finite gyroradius corrections. As examples we consider three different turbulence models namely the slab model, a noisy slab model, and the two-dimensional model.

The remainder of this paper is organized as follows. In Section 2 we briefly discuss the relation between particle and guiding center coordinates and the corresponding diffusion coefficients. Quasi-linear theory is employed in Section 3 and in Section 4 we use a more advanced approach based on non-linear diffusion theory. We end with a short summary and some conclusions in Section 5.

2. Relation between guiding center and particle coordinates

Usually one is interested in the coordinates of the charged particle interacting with turbulence. In the following we refer to these coordinates as *particle coordinates* and we will use the symbols $\vec{x}$ and $\vec{v}$ for particle position and velocity, respectively. Alternatively, one can use the coordinates $\vec{R}$ defined via (see, e.g., Schlickeiser (2002)).

$$\vec{R} = \vec{x} + \frac{c}{qB_0}(\vec{p} \times \vec{e}_z) = \vec{x} + \frac{1}{\Omega}(\vec{v} \times \vec{e}_z) \tag{2}$$

where $\Omega = (qB_0)/(mc\gamma)$ is the unperturbed gyrofrequency. Here we have used the electric charge of the particle q, the rest mass m, the speed of light c, and the Lorentz factor γ. For the velocity $\vec{v}$ of the particle we use spherical coordinates

$$\begin{aligned} v_x &= v\sqrt{1-\mu^2}\cos\Phi \\ v_y &= v\sqrt{1-\mu^2}\sin\Phi \\ v_z &= v\mu \end{aligned} \tag{3}$$

with the particle speed v, the pitch-angle cosine μ, and the gyrophase Φ.

In the unperturbed case, where we have by definition $\delta\vec{B} = 0$, the vector $\vec{R}$ corresponds to the position of the guiding center. Therefore, we call $\vec{R}$ the *guiding center coordinates*. To obtain the velocity of the guiding center, we consider the time derivative of Eq. (2). This gives us

$$\begin{aligned} \vec{V} := \frac{d\vec{R}}{dt} &= \vec{v} + \frac{c}{qB_0}\left(\frac{d\vec{p}}{dt} \times \vec{e}_z\right) = \vec{v} + \frac{1}{B_0}\left[\left(\vec{v} \times \vec{B}\right) \times \vec{e}_z\right] \\ &= v_z\frac{\vec{B}}{B_0} - \vec{v}\frac{\delta B_z}{B_0}. \end{aligned} \tag{4}$$

where we have employed the *Newton–Lorentz equation* $d\vec{p}/dt = q(\vec{v} \times \vec{B})/c$ and the *Graßmann Identity*

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}(\vec{a}\cdot\vec{c}) - \vec{c}\left(\vec{a}\cdot\vec{b}\right). \tag{5}$$

Therefore, we can express the components V_i of the guiding center velocity vector by the components of the particle velocity vector v_i and the magnetic field components. Very often in diffusion theory, turbulence models with $\delta B_z = 0$ are considered. Examples are the slab and the two-dimensional model (see below for a definition of these two models). In this particular case Eq. (4) simplifies to

$$V_x = v_z\frac{\delta B_x}{B_0}, \quad V_y = v_z\frac{\delta B_y}{B_0}, \quad \text{and} \quad V_z = v_z. \tag{6}$$

From a more practical point of view, these equations are valid as long as the condition $\delta B_z \ll B_0$ is satisfied. In this case they can be used as starting point to compute the perpendicular diffusion coefficient.

We have to be very careful if *particle coordinates* or *guiding center coordinates* are used. The guiding center coordinates (X, Y, Z) are related to particle coordinates (x, y, z) via

$$X = x + \frac{v_y}{\Omega}, \quad Y = y - \frac{v_x}{\Omega}, \quad \text{and} \quad Z = z \tag{7}$$

as derived from Eq. (2).

The perpendicular diffusion coefficients can be calculated as time derivatives of the corresponding mean square displacements. From Eq. (7) we find the following relations