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# Central difference predictive filter for attitude determination with low precision sensors and model errors

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**Abstract**

Attitude determination is one of the key technologies for Attitude Determination and Control System (ADCS) of a satellite. However, serious model errors may exist which will affect the estimation accuracy of ACDS, especially for a small satellite with low precision sensors. In this paper, a central difference predictive filter (CDPF) is proposed for attitude determination of small satellites with model errors and low precision sensors. The new filter is proposed by introducing the Stirling's polynomial interpolation formula to extend the traditional predictive filter (PF). It is shown that the proposed filter has higher accuracy for the estimation of system states than the traditional PF. It is known that the unscented Kalman filter (UKF) has also been used in the ADCS of small satellites with low precision sensors. In order to evaluate the performance of the proposed filter, the UKF is also employed to compare it with the CDPF. Numerical simulations show that the proposed CDPF is more effective and robust in dealing with model errors and low precision sensors compared with the UKF or traditional PF.


*Keywords:* Attitude determination; Central difference predictive filter; Model error; UKF; PF

## 1. Introduction

Over the past decades, there has been a growing interest in small satellites due to their low cost and short development period (Tsao and Liu, 2009; Carmi and Oshman, 2006). Much effort has been made to explore the possible use of small satellites for various space missions, as well as to develop small satellite technologies. Attitude determination is fundamental to the Attitude Determination and Control System (ADCS) of a small satellite with high performance. However, low-complexity, low-cost and low-precision sensors are usually adopted for small satellites because of the limitations on cost, size, mass and power, which sometimes greatly hinder the accuracy of the attitude determination of the satellite. Especially when an attitude dynamic model is used to instead of an inertial gyro to estimate the angular velocity, model errors are inevitably introduced.

In addition to the uncertainties associated with the physical properties of the satellite, various perturbations in space cannot also be modeled accurately, which leads to the existence of model errors. Limited by the present level of technology, the model errors cannot be estimated/predicted accurately. Hence, the model errors and low precision sensors have become the main challenges for the

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attitude determination of small satellites. To address these problems and satisfy the demands of future missions, research on the study of filter algorithms for attitude determination with low precision sensors in the presence of model errors is necessary.

To address these problems, several studies have considered the extended Kalman filter (EKF) (Lefferts et al., 1982; Markley, 2003) and unscented Kalman filter (UKF) (Hongmei, 2004; John and Markley, 2003). EKF has been widely used for nonlinear estimation problems, including satellite attitude determination. However, its performance depends not only on the accuracy of the system models, but also on the prior statistical knowledge of the noise. It is known that the statistical characteristics of the noise are difficult to obtain due to the complex environment (Ding et al., 2007; De Ruiter, 2010). Besides, EKF has no capability to adapt itself to the model errors. In addition, the UKF is proposed based on the Unscented Transformation (UT) technique to extend the Kalman filter. This is done by using a minimal set of deterministically chosen sigma points, which match the mean and covariance of a probability distribution. When these sigma points are propagated through the nonlinear function, they capture the posterior mean and covariance accurately to higher accuracy. But it also depends on the prior statistical knowledge of the noise, and has poor ability to deal with the model errors. Crassidis and Markley (1997a,b), Crassidis and Landis (1997) presented a real-time nonlinear filtering technique named predictive filter (PF), which has good ability to cope with model errors and has attracted wide attention. This algorithm has been successfully applied to numerous situations, including spacecraft attitude determination (Crassidis and Markley, 1997a,b; Crassidis, 1999; Lin and Deng, 2001), chaotic synchronization system (Ji and Yang, 2010), etc. However, the estimate of model error for PF captures the posterior mean accurately only to the first order; therefore, it will inevitably bring some problems, such as the loss of the estimate precision, slow rate of convergence, etc. If the estimate of the model error deviates largely from its actual value, this error will be propagated by the state equations to amplify the error effect, resulting in filter divergence (Gong and Fang, 2008). In addition, the filtering performance will degrade with small simulation step size. Due to the problems discussed above, the development and application of PF have been relatively limited.

In light of the above-mentioned issues, an alternative is needed. The technique of Stirling's polynomial interpolation formula (Froberg, 1972) provides an effective strategy to address these issues. The central difference predictive filter (CDPF) is proposed here and implemented by using the second-order Stirling's polynomial interpolation formula to calculate the mean and covariance of the model error and the system state. This derivativeless method, based on predictive filter consistently outperforms the earlier PF, not only in terms of estimation accuracy, but also in filter robustness and ease of implementation. In this paper, the estimated model error for the CDPF is demonstrated to capture the posterior mean accurately up to the 3rd order for nonlinear Gaussian system, with errors only introduced in the 4th and high orders. At the same time, the estimated model error can capture the posterior mean accurately to the 2nd order for any nonlinearity, which is better than the accuracy of the classical PF. Furthermore, it is also proved in the paper that the estimation accuracy of the system states is better than that of PF. To verify the performance of the CDPF, several scenarios with different levels of the model error and measurement noise are simulated. The UKF is also employed to compare it with the proposed CDPF.

Organization of this paper proceeds as follows: in Section 2, the statement of the problem is presented and the attitude determination equations of motion are reviewed. In Section 3, the estimation accuracy of the model error and system state is analyzed for CDPF, which shows that the estimation accuracy of the model error and system state as well as of the covariance is all higher than traditional PF. In addition, the general algorithm flow of the CDPF is also described. In Section 4, numerical simulations are presented to illustrate the performance of the proposed CDPF in comparison with the PF and UKF in order to verify the theoretical results of Section 3. Some main conclusions are drawn in the final section.

## 2. Review of attitude motion equations

In this section, an attitude determination of a small satellite is first reviewed, which utilizes the measurements of a three-axis magnetometer and a sun sensor with low precision. In addition, the attitude dynamics model is adopted to replace the inertial gyro to determine the angular velocity, which inevitably introduces model errors. Due to the existence of system uncertainties and external disturbances from the space, it is really difficult to design a satellite attitude determination system very accurately, which greatly hinders the attitude estimation performance. Especially for the satellites operating for a long term in an orbit, the serious perturbation or degradation of sensor accuracy will have detrimental effects on the attitude estimation performance. Therefore, various kinds of perturbations can be included in the total model error, which need to be considered in the attitude dynamics model.

The attitude dynamic model without gyro can be expressed by

$$\begin{aligned}\dot{\boldsymbol{X}} &= \begin{bmatrix}\dot{\boldsymbol{q}}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})\boldsymbol{q}\\ \boldsymbol{J}^{-1}[\boldsymbol{N}-\boldsymbol{\omega}\times(\boldsymbol{J}\omega)]+\boldsymbol{J}^{-1}\boldsymbol{d}\end{bmatrix}\\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{X}) &= \begin{bmatrix}\boldsymbol{C}_i^b(\boldsymbol{q})S_1\\ \boldsymbol{C}_i^b(\boldsymbol{q})B_2\end{bmatrix} + \begin{bmatrix}\Delta v_1\\ \Delta v_2\end{bmatrix}\end{aligned} \tag{1}$$

where

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix}0 & -\boldsymbol{\omega}^T\\ \boldsymbol{\omega} & -[\boldsymbol{\omega}\times]\end{bmatrix},\quad [\boldsymbol{\omega}\times] = \begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix} \tag{2}$$