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Two-dimensional phase plane structure and the stability of the orbital motion for space debris in the geosynchronous ring

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Abstract

Based on the orbital resonance model, we study the two-dimensional phase plane structure of the motion of space debris orbiting the geosynchronous ring under the combined effects of the tesseral harmonics J_{22}, J_{31} and J_{33} of the Earth's gravitational field. We present the main characteristic parameters of the two-dimensional phase plane structure. We also analyze the stability of the two-dimensional phase plane structure with numerical method. Our main findings indicate that the combined effects of the tesseral harmonics J_{22}, J_{31} and J_{33} fully determine the two-dimensional phase plane structure of the space debris, and it remains robust under the effect of the Earth's actual gravitational field, the luni-solar perturbations and the solar radiation pressure with the normal area-to-mass ratios.

Keywords: Space debris; Geosynchronous orbits; Resonance

1. Introduction

A distinctive dynamical phenomenon of the space debris orbiting the geosynchronous ring is orbital resonance with the Earth rotation. Its principal resonant term arises from the tesseral harmonic J_{22} associated with the Earth's equatorial ellipticity causing libration of the space debris orbiting the geosynchronous ring around the nearest stable position either about 75°E or about 105°W (Blitzer et al., 1962; Schildknecht, 2007). The Ideal Resonance Problem of the uncontrolled satellites (or space debris) orbiting the geosynchronous ring was first developed by Garfinkel (1966) with the principal zonal harmonic J_2 and tesseral harmonic J_{22} of the Earth's gravitational field only. Many scientists studied this orbital resonance problem arising from tesseral harmonic J_{22} (Jupp, 1969; Dallas and Diehl, 1977; Nacozy and Diehl, 1982; Liu et al., 1991; Zhao and Liu, 1991; Valk et al., 2009). Liu et al. discussed the two-dimensional (2D) phase plane structure and its main characteristic parameters of the 1:1 resonance of the satellites orbiting the geosynchronous ring based on the ideal resonance model in detail, and presented the analytical expressions of the libration period and libration width of the 1:1 resonance of the satellites (Liu et al., 1991; Zhao and Liu, 1991).

However, because of asymmetry of the Earth's gravitational field due to the higher degree and order tesseral harmonics, especially the third degree terms, the space debris (or uncontrolled satellites) orbiting the geosynchronous ring can also move in the way of complex ('long') libration around both stable positions (we can call it double libration) besides the way of simple libration around the nearest

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stable position (either about 75°E or about 105°W) (Flohrer et al., 2011). Vashkov'yak and Lidov first found the effect of splitting separatrix of the geostationary satellite under the influence of the higher degree and order tesseral harmonics of the Earth's gravitational field, which leads to the appearance of the libration zone including both stable positions; and obtained the estimates of the libration periods in the regions surrounding stable positions: 740 days in the neighborhood of the position with a longitude of 75°E, 930 days in the neighborhood of the position 105°W and the minimal libration period with respect to both stable positions 2900 days (Vashkov'yak and Lidov, 1973; Kuznetsov and Kaiser, 2007). The Ideal Resonance Problem of the objects orbiting the geosynchronous ring cannot explain this double libration phenomenon. Giacaglia presented an Extended Resonance Problem exhibiting the case of double libration (Giacaglia, 1970). Garfinkel defined another form of an Extended Resonance Problem with the phase-plane trajectories exhibiting a double libration, constructed the first order solution for the case of double libration by a generalization of the procedure used in solving the Ideal Resonance Problem, and presented a Generalized Ideal Resonance Problem with the double libration illustrated by the horseshoe-shaped orbits enclosing two libration centers (Garfinkel, 1975, 1976). These Extended Resonance Problems or Generalized Ideal Resonance Problem stated in Giacaglia (1970); Garfinkel (1975, 1976) are generalized theoretical analyses and were not applied to the actual motion of the objects orbiting the geosynchronous ring. Lara and Elipe generated families of planar periodic orbits emanating from the geostationary points (both stable and unstable) and showed that even for the unstable points, it is possible to have stable periodic orbits, which also reflects the existence of the double libration (Lara and Elipe, 2002).

Exactly because the space debris orbiting the geosynchronous ring has the double libration regime besides simple libration (Flohrer et al., 2011), while its conventional ideal resonance model with only a resonant term arising from tesseral harmonic J_{22} cannot explain this regime, it is necessary to establish a new ideal resonance model to better characterize its 2D phase plane patterns of the motion. Inspired by Garfinkel's idea: if the sine of an odd multiple of resonant variable x is added into the conventional ideal resonance model, the topological picture exhibits the novel feature of double libration enclosing two libration centers (Garfinkel, 1975), an extended orbital resonance model of the motion of space debris orbiting the geosynchronous ring is established in this paper by considering the combined effects of the zonal harmonic J_2 and tesseral harmonics J_{22}, J_{31} and J_{33} of the Earth's gravitational field. Based on this extended orbital resonance model, the 2D phase plane structure (in which the double libration regime is included) and its main characteristic parameters are investigated. Under the effect of the Earth's actual gravitational field, the luni-solar perturbations and the solar radiation pressure with the normal area-to-mass ratios (A/M) less than $0.02m^2/kg$, the stability of the 2D phase plane structure is analyzed with numerical method.

2. An extended orbital resonance model with J_{22}, J_{31} and J_{33}

The Earth's gravitational potential V may be expressed in terms of geocentric distance r, latitude φ and geographic longitude λ, as

$$\begin{aligned} V &= -\frac{\mu}{r}\left[1+\sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{a_e}{r}\right)^n P_{nm}(\sin\varphi)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right] \\ &= -\frac{\mu}{r}\left[1-\sum_{n=2}^{\infty}J_n\left(\frac{a_e}{r}\right)^n P_n(\sin\varphi)\right. \\ &\quad \left. +\sum_{n=2}^{\infty}\sum_{m=1}^{n}J_{nm}\left(\frac{a_e}{r}\right)^n P_{nm}(\sin\varphi)\cos m(\lambda-\lambda_{nm})\right], \end{aligned} \tag{1}$$

where $\mu = G_N M_e$ is the product of the Newtonian constant of gravitation by the mass of the Earth, a_e is the Earth mean equatorial radius, C_{nm} and S_{nm} are the spherical harmonic coefficients of degree n and order m of the Earth's gravitational field, P_n, P_{nm} are the Legendre polynomial and associated Legendre function respectively, $J_{nm} = \sqrt{C_{nm}^2 + S_{nm}^2}, J_n = -C_{n0}$, and $m\lambda_{nm} = \arctan(S_{nm}/C_{nm})$.

Introduce the following canonical variables

$$\begin{cases} L = \sqrt{\mu a} & l = M + \omega + \Omega - \theta_G, \\ G = \sqrt{\mu a(1-e^2)} - L & g = \omega, \\ H = \sqrt{\mu a(1-e^2)}\cos i - L & h = \Omega, \end{cases} \tag{2}$$

which are linear combinations of the Delaunay canonical variables, where a, e, i, ω, M and Ω are the classical orbital elements (Ω is measured from the equinox), θ_G is the Greenwich hour angle satisfying $\dot{\theta}_G = n_e$, and n_e is the rotational angular velocity of the Earth.

We first focus on the orbital resonance problem due to geopotential perturbation here, and leave the luni-solar perturbation effects for the space debris orbiting the geosynchronous ring aside. If only the secular and resonant terms of the dominant zonal harmonic J_2 and main tesseral harmonics J_{22}, J_{31} and J_{33} are considered, by averaging short periodic terms and neglecting the second order effects due to J_2, the Hamiltonian function of the 1:1 orbital resonance problem of the space debris orbiting the near circular geosynchronous ring can be expressed in the canonical variables Eq. (2) as

$$\begin{aligned} F &= \frac{\mu^2}{2L^2} + n_e L + \frac{\mu^4 a_e^2}{L^6}J_2\left(\frac{1}{2}-\frac{3}{4}\sin^2 i\right) \\ &\quad + \frac{\mu^4 a_e^2}{L^6}J_{22}f_{220}(i)\cos 2(l-\lambda_{22}) + \frac{\mu^5 a_e^3}{L^8}J_{31}f_{311}(i) \\ &\quad \times \cos(l-\lambda_{31}) + \frac{\mu^5 a_e^3}{L^8}J_{33}f_{330}(i)\cos 3(l-\lambda_{33}), \end{aligned} \tag{3}$$

where $f_{220}(i), f_{311}(i)$ and $f_{330}(i)$ are inclination functions and defined as (Kaula, 1966)