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Applications of the admissible region to space-based observations

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Abstract

Space-based optical sensors for space situational awareness allow for tracking of objects that are smaller than currently capable with ground-based systems with less influence of atmospheric phenomena. Multiple measurements must be combined in order to obtain a full state description, but their association is uncertain *a priori.* In this paper, the direct Bayesian admissible region method is applied to space-based observations. Simulation results for both a low-Earth (LEO) and geosynchronous orbit (GEO) observer are shown. For the former, the proposed method successfully associates 101 observations of 8 Earth-orbiting objects taken over the span of 48 h. For the latter, lack of relative motion is identified as a key difficulty in the association process. This problem can be remediated by improved observation strategies.

Keywords: Space situational awareness; Too-short-arc problem; Space-based observations; Admissible region; Probability density function

1. Introduction

Situational awareness of Earth-orbiting objects such as active satellites and space debris is highly important for future human activities in space. Currently, objects in medium Earth orbits (MEO) and geosynchronous orbits (GEO) are mostly observed by ground-based optical sensors which can detect objects on the order of 10^1 centimeters in diameter at these altitudes (Rossi, 2005). Space-based observations allow for tracking of smaller objects regardless of weather, and several missions have been flown to date (Vallado, 2010). In 1997, the Space Based Visible (SBV) sensor on the Midcourse Space Experiment (MSX) satellite became a contributing sensor to the Space Surveillance Network (SSN) after the completion of its primary mission for the Ballistic Missile Defense Organization (Sharma et al., 2002; Stokes et al., 1998). The Space Based Surveillance System (SBSS) launched in 2010 is a follow-up satellite to MSX/SBV dedicated to the task of space situational awareness (SSA)[2] (The Boeing Company, 2011). Space agencies, corporations, and academia in Europe and Asia have also shown interest in space-based SSA (Flohrer et al., 2011; Tagawa et al., 2011; Hongzheng et al., 2012). Outside the realm of dedicated payloads, hosted payload sensors on spacecraft in geostationary or geostationary transfer orbits may provide observational capabilities at a much lower cost (Lowe et al., 2010; Vallado et al., 2011; Hackett, 2012; Shell, 2011).

For optical observations of Earth-orbiting objects, either ground- or space-based, state information is expressed in terms of a time history of angles. That is, the range and range-rate remain largely unknown. Therefore, to determine the orbit of the observed object, multiple observations must be combined. It is generally uncertain, however, whether two arbitrary tracks are of the same object. This is the crux of the *too short arc* (TSA) problem (Tommei et al., 2007; Milani et al., 2004, 2005, 2012; Farnocchia et al., 2010; DeMars and Jah, 2012). Fujimoto and Scheeres have proposed an association technique for

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ground-based observations which uses a probability density function (pdf) on the range and range-rate of an observed object, or the *admissible region* (AR), defined based on some physical constraints of the possible orbits (Maruskin et al., 2009; Fujimoto and Scheeres, 2012a). By applying Bayes' rule to multiple AR pdfs, one can associate their corresponding optical observations as well as give an initial state estimate.

In this paper, the direct Bayesian admissible region method is applied to space-based optical observations of Earth-orbiting objects. First, the AR, as well as other necessary concepts, are introduced and defined mathematically (*Background*). Next, examples of processing simulated optical observations of objects in both MEO and GEO from an observer in either low Earth orbit (LEO) or GEO are examined (*Examples*). For space-based observations, the observer's state is always a solution, albeit degenerate. This issue is addressed by excluding a small region about the observer's state in the AR. The LEO observer case works well, but the GEO observer case experiences numerical issues due to lack of relative motion; similar issues are discussed in existing research on GEO-on-GEO space-based tracking (Vallado, 2010; Tombasco, 2011; Woffinden and Geller, 2008). Therefore, from an initial orbit determination perspective, if one is observe geostationary objects from the geosynchronous region, both a slightly eccentric and inclined orbit ($e \sim 10^{-3}, i \sim 10^{-1}\circ$) and separating the observations by about 10 h are beneficial. Finally, the effect of observation error to the AR technique is discussed quantitatively (*Errors in the Admissible Region Map*). Errors in the observations as well as any dynamical modeling errors impose a definite upper bound on the accuracy of state estimation. This paper sets the theoretical groundwork for associating space-based observations and obtaining an initial orbit estimate via the AR method.

2. Background

2.1. Attributable vector for space-based observations

Regardless of where the observation was made, for optical-only observations of Earth-orbiting objects, the measured state variables are the object's apparent angles plus their time stamps (Maruskin et al., 2009). That is, the observer-centric range and range rate remain largely unconstrained. Thus, each track can be mathematically expressed in terms of an attributable vector $\mathcal{A}$ at epoch t of the observation (Milani et al., 2004; Tommei et al., 2007):

$$\mathcal{A} = (\alpha, \delta, \dot{\alpha}, \dot{\delta}) \in [-\pi, \pi) \times (-\pi/2, \pi/2) \times \mathbb{R}^2, \tag{1}$$

where α and δ specify the observer-centric angular position of the object. A discussion of how one may estimate an attributable vector from a given track of data, in particular the time derivatives of the angles, can be found in Maruskin et al. (2009). J2000 is chosen as the coordinate system in this paper so that α is the right ascension and δ is the declination, but note that one is free to choose any coordinate system.

For a more general description of the track suitable for space-based observations, the geocentric position $\mathbf{P}_O$ and velocity $\dot{\mathbf{P}}_O$ of the observation point are appended in addition to the epoch time t_0:

$$\mathfrak{X} = (\mathcal{A}, t_0, \mathbf{P}_O, \dot{\mathbf{P}}_O). \tag{2}$$

In this paper, perfect knowledge regarding the state of the observing satellite is assumed. This assumption allows us to use the same formulation of the attributable vector as for ground-based observations except for the kinematics of the observation point. Qualitatively, this assumption is expected to be good especially when the observer is in a LEO orbit because the errors in the observer's state (up to 10^1 m-level in position, 10^0 mm/s-level in velocity Vallado, 2007, 2010) is much less than the uncertainty in range/range-rate. A quantitative discussion is given in Section 4.

2.2. The admissible region

For any attributable vector $\mathfrak{X}$, we can take different values of range and range-rate $(\rho, \dot{\rho})$ to complete the observer-centric spherical coordinates of the object and thus obtain different physical orbits. However, not all of these orbits are relevant for any given application. For instance, we may not be interested in objects that escape Earth orbit a couple of hours after observation. Rather, a closed region of the $(\rho, \dot{\rho})$ plane can be defined such that all of the physically relevant orbits are contained within the interior of this region. That is, each interior point is a potential estimate of the observed object's state with some assigned probability. We define a uniform probability density function (pdf) over this domain as the *admissible region* $F_{\mathfrak{X}}(t)$, which is a function of the observation time t, and each point in the domain as a *virtual particle* (VP) (Milani et al., 2004). A set of criteria $\mathcal{C}$ defining the AR domain for ground-based observations have been proposed by Maruskin et al. (2009) and DeMars and Jah (2012). For this paper, we modify the criteria as follows:

$$\mathcal{C} = \bigcap_{i=1}^{4} \mathcal{C}_i \tag{3}$$

and

$$\mathcal{C}_1 = \{(\rho, \dot{\rho}) : E \leqslant 0\} \quad \mathcal{C}_2 = \{(\rho, \dot{\rho}) : 0 \leqslant \rho \leqslant 14\}, \tag{4}$$

$$\mathcal{C}_3 = \{(\rho, \dot{\rho}) : 1.03 \leqslant r_p\} \quad \mathcal{C}_4 = \{(\rho, \dot{\rho}) : r_a \leqslant 25\}, \tag{5}$$

where E is the specific geocentric energy, and r_p and r_a are the perigee and apogee radii of the observed object, respectively. Units of length are in Earth radii (r_E). Compared to existing work, the main difference is that the range of observer-centric range values now includes objects that are very close to the observer. Nevertheless, the criteria still encompass objects relevant in SSA as well as filter out those with extremely high eccentricity. Fig. 1 is an example of an AR.

The AR expresses our *limited* knowledge regarding the directly unobserved variables ρ and $\dot{\rho}$. In conventional fil-