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Lunar gravity gradiometry and requirement analysis

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Abstract

The current lunar gravity field models are mostly concluded from lunar exploration programs either by observations of satellite orbit perturbations or by intersatellite ranging and ranging rate measurements between two low-orbiting spacecrafts. Especially, the measurements from the Gravity Recovery and Interior Laboratory (GRAIL) have been used to produce the Moon's gravity field models with spatial resolutions up to spherical harmonic degree 420 and 660 for the primary mission and extended mission, respectively. This paper presents a method using satellite gravity gradiometry, which not only can determine a moon's gravity field model with a higher resolution, but also can obtain the medium and short wavelength components information with a higher accuracy. This advantage is still obvious in comparison with the results from GRAIL's extended mission even though the maximum degree of the model derived from the mission is close to that achieved by the satellite gradiometry discussed in this study. Based on the simulations and analysis, a mission with the hypothesis of an orbit height of about 20 km, a mission duration of about 14 days, and a gradiometer accuracy level of about 30 $mE/\sqrt{Hz}$ is proposed, and it permits determination of a lunar gravity field model with a high accuracy of 14 mGal and a geoid with an accuracy of 20.5 cm, both at a spatial resolution of 7 km, corresponding to spherical harmonic degree and order 789. The effects of gravity gradient measurement errors, orbit height, tracking accuracy, mission duration and sampling rate are analytically investigated by the direct relationship between the satellite gravimetry measurements and coefficients of the Moon's gravitational potential, which is verified by the least-squares method.

Keywords: Lunar exploration; Gradiometry; Error analysis; Gravity field

1. Introduction

The Moon's gravity field and its variations reflect the Moon's density structure, mass distribution and interiors dynamics (Binder, 1998; Sagitov et al., 1986) see e.g. The investigation of the Moon's gravity field was started in 1966 with the Russian Luna 10 mission and was followed by the first US Lunar Orbiter (LO-I) in the same year (Konopliv et al., 2001; Ferrari, 1977; Bills and Ferrari, 1980). The current best lunar global gravity field models LP165P and SGM100 are developed from the tracking data of the 1998–1999 Lunar Prospector (LP) mission (Konopliv et al., 1998) and the 2007–2009 SELENE mission (Liu et al., 2010; Kikuchi et al., 2009), respectively. LP165P provides the lunar geoid with an accuracy of about 12.7 m at a half wavelength resolution of 33 km, and SGM100 predicts the lunar geoid with an accuracy of about 4.5 m at a half wavelength resolution of 55 km. By using low-low satellite-to-satellite tracking data (ll-SST, GRACE-like), the GRAIL's primary mission and extended mission produces new models of the Moon's gravity field with greatly improved accuracy and spatial resolution up

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to degree 420 and 660, respectively Zuber et al. (2012), Zuber (2013) and Hoffman (2009).

Up to now, all of the launched lunar gravimetry missions are based either on observations of satellite orbit perturbations or on intersatellite ranging and ranging rate measurements between two low-orbiting spacecrafts. These measurements can derive the long wavelength components of the Moon's gravity field with higher accuracy than the medium and short wavelength components (Iwata et al., 2002; Zuber et al., 2012). Since synchronous Moon rotation prohibits a direct link between a ground tracking station and a spacecraft over the farside of the Moon, the details of farside lunar gravity models from the missions, whose observations are satellite orbit perturbations obtained on the Earth, are generally limited, except for SELENE mission that relys on a relay subsatellite for detecting the lunar farside data (Kawano et al., 2010). In addition, for all the launched lunar gravimetry missions, unlike in the Earth's gravity field recovery missions, e.g. CHAMP and GRACE missions, non-gravitational forces such as solar radiation pressure and mechanical forces were not directly measured by an accelerometer, but were given through mathematical models. This would degrade the accuracy and resolution of the Moon's gravity field models (Flury et al., 2008).

In this paper, a lunar satellite gravity gradiometry mission is proposed for improving the Moon's gravity field model. The application of superconducting gravity gradiometer to a lunar gravity field mapping mission has been discussed by Paik et al. (Paik, private communication, 2011). In this proposal, a gradiometer based on similar techniques that was used in GOCE mission is considered due to its simple structure and past space mission experience (European Space Agency, 1999). In comparison with all the launched missions, the overriding advantage of the gradiometry is the more precise determination of the medium and short wavelength gravity components. The gradiometry can derive the gravity gradient components by combining the differential mode accelerations, and can infer the non-gravitational forces by the common mode accelerations. As a result, these measurement modes are not only effective in recovering the medium and short wavelength gravity field information, but are also useful in reducing the effect of non-gravitational forces in improving the gravity field recovery accuracy. Moreover, the lunar gradiometry could relax the requirements of satellite orbit determination, and could improve the resolution and accuracy of the gravity field of lunar farside region.

The paper is organized as following. In Section 2, the lunar gradiometry principle is briefly explained. In Section 3, a direct relationship between the power spectral density (PSD) of the satellite gravimetry measurements and coefficients of the gravitational potential is established (Cai et al., 2012). This relationship is verified by the least-squares (LS) method in the following section. The resolution and accuracy of gravity field recovery from a lunar gradiometry mission with different parameters are simulated and discussed. Through comparison of different parameters, a mission scenario with an orbit height of about 20 km and a gradiometer accuracy level of about 30 $\mathrm{mE}/\sqrt{\mathrm{Hz}}$ is recommended. Finally, the paper concludes with a conclusion.

2. Lunar gravity gradiometry principle

Gravity gradients are the second order derivatives of the gravitational potential V, denoted as $\partial^2 V/\partial x_i \partial x_j = V_{ij}$ $(i, j = 1, 2, 3)$, which are sensitive to the medium and short wavelength parts of the gravity field. It can be expressed as a second-rank tensor with nine components with respect to a local north-oriented frame (Rummel and Colombo, 1985)

$$V_{ij} = \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}, \tag{1}$$

where x points towards the north, y points towards the east, and z points upwards in selenocentric radial direction. Since the gravitational potential obeys Laplace's equation, V_{ij} is symmetric, and only five components out of the nine components of the gradient tensor are independent. The gradient tensor components are precisely measured by the gradiometer with other auxiliary instruments on board satellite, and the Moon's gravity field could be derived using these measurements (Rummel et al., 1993).

In analogous to the Earth's physical geodesy theory, the Moon's gravity field can be expanded into a series of spherical harmonics (Sagitov et al., 1986), and the spherical harmonics representation for the Moon's disturbance potential T is written as

$$T(r,\theta,\lambda) = \frac{GM}{R}\sum_{l=2}^{\infty}\left(\frac{R}{r}\right)^{l+1}\sum_{m=0}^{l}\bar{P}_{lm}(\cos\theta)(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda), \tag{2}$$

where

r, θ, λ selenocentric spherical coordinates (radius, colatitude, longitude);
R reference length (mean semi-major axis of the Moon);
GM gravitational constant times mass of the Moon;
l, m degree, order of spherical harmonic;
$\bar{P}_{lm}(\cos\theta)$ fully normalized Legendre functions;
$\bar{C}_{lm}$, $\bar{S}_{lm}$ fully normalized dimensionless potential coefficients.

The potential coefficients $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are the unknown coefficients that should be determined from mission observations as the solution of the lunar gravity field recovery.

3. Spectral analysis of the Moon's gravity field

It is an effective approach to determine the mission parameters by investigating the direct relationship between