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Transfers from Earth to Earth–Moon L_3 halo orbits using accelerated manifolds

Kathryn Davis [a,*], Jeffrey Parker [a], Eric Butcher [b]

[a] *The Colorado Center for Astrodynamics Research, University of Colorado, Boulder, CO 80309, USA*
[b] *Aerospace and Mechanical Engineering, University of Arizona, Tuscon, AZ 85721, USA*

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Abstract

This paper is concerned with two-impulse transfers from Earth to Earth–Moon L_3 halo orbits. After an orbit injection maneuver from an Earth orbit, a spacecraft travels on a ballistic accelerated manifold trajectory to a position intersection with a halo orbit where an orbit injection maneuver is executed. Although many types of transfers are located, our primary concern is transfers that require either a low transfer time of flight or a small orbit injection maneuver. Several families of transfers lie along the edge of a time of flight/injection maneuver Pareto Front. These families share similar characteristics and are shown to be an extension of a transfer that utilizes a stable invariant manifold. The quickest family of transfers to L_3 can be completed in 28.5–33 days with an injection maneuver of 61.75–130 m/s, with shorter duration transfers requiring a larger injection maneuver. The family of transfers with the smallest injection maneuvers given a duration limit of 140 days required 13.45 m/s.

Keywords: Dynamical systems theory; Invariant manifolds; Libration point orbits; Orbit transfers; Three-Body trajectories

1. Introduction

The dynamics of spacecraft on libration point orbits (LPOs) and their associated invariant manifolds present many interesting opportunities for mission design. In the last four decades, missions such as ISEE-3, Herschel Space Observatory, Genesis, and ARTEMIS have traveled to and operated in LPOs about the collinear points L_1 and L_2 in the Earth–Moon and Sun–Earth systems. These missions have a diverse array of objectives; studying the Universe by the light of the far-infrared and submillimeter portions of the spectrum, observing and entrapping solar wind particles and returning them to Earth, and analyzing the effect of the Sun's radiation at our Moon.

Presently, missions have only orbited the libration points L_1 and L_2. However, there has been an increased interest in orbits about Earth–Moon L_3 due to their advantageous geometry. For example, many orbits at L_3 have the ability to view almost 50% of the Earth's surface. It has recently been shown that a constellation of three spacecraft (two in Earth–Moon L_3 LPOs and one in an L_1 LPO) can provide almost complete visibility of the Earth's surface (Davis et al., 2013). This constellation could also provide continuous coverage of geosynchronous regions. Orbits at L_3 are relatively stable and would likely require minimal station-keeping (Parker et al., 2014). Furthermore, satellites traveling to an L_3 orbit require substantially less fuel than those traveling to geostationary orbit.

In the past, invariant manifolds have been used to construct transfers from Earth to LPOs in both the Sun–Earth (SE) system, e.g., Genesis (Lo et al., 2001) and the Earth–Moon (EM) system, e.g., ARTEMIS (Broschart et al.,

* Corresponding author.
E-mail addresses: kate.davis@Colorado.edu (K. Davis), parkerjs@Colorado.edu (J. Parker), ebutcher@email.arizona.edu (E. Butcher).

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2009). There are many examples in literature of methods to build these transfer trajectories (Howell et al., 1994; Koon et al., 2000; Parker and Born, 2008).

1.1. Trajectories to Sun–Earth L_3 orbits

Designing a trajectory to the vicinity of Sun–Earth L_3 is particularly challenging since the region lies on the opposite side of the Sun as the Earth with almost the same Sun–Earth distance. One suggested transfer method is to use invariant manifold trajectories associated with L_3 LPOs. However, the invariant manifolds of L_3 Lyapunov orbits may take thousands of years to leave the vicinity of L_3, never closely approach Earth, and are thus impractical for mission design (Hou et al., 2008).

Another suggested approach is to utilize the unstable invariant manifolds of L_1 and L_2 LPOs since they travel close to Earth and to the vicinity of L_3. These transfers require over nine years of transfer time. Tantardini compared several methods for reaching L_3 orbits (Tantardini, 2009). Bi-elliptic transfers, transfers using patched conics with gravity assists, and transfers utilizing low-thrust were modeled using the two-body problem. Following the work done by Hou et al., transfers using invariant manifolds associated with Lyapunov orbits about L_1 and L_2 were also explored (Tantardini et al., 2010).

1.2. Trajectories to Earth–Moon L_3 orbits

Recently, Larsen et al. modeled transfers from a 300 km low Earth orbit (LEO) to the EM L_3 point by employing the exterior stable manifold of L_3 that lies in the x-y plane (Larsen et al., 2012). The transfer, optimized for minimum fuel, requires an impulsive burn to leave LEO and then performs a second impulsive maneuver to transfer onto the stable manifold. The spacecraft then travels along the exterior stable manifold and arrives at L_3. The first and second ΔVs are 3.07 km/s and 0.41 km/s, respectively, for a total ΔV of 3.48 km/s. The transfer time is approximately 200 days.

Haapala et al. constructed sample missions that included orbits at or near L_3, including a tour of the L_3, L_4, and L_5 libration points using resonant orbits as the transfer mechanism, and a transfer between orbits in the vicinity of L_2 and L_3 employing manifolds associated with an L_2 vertical orbit (Haapala et al., 2013).

Davis et al. introduced the concept of the pseudo-manifold for transfers between low-Earth orbit (LEO) and L_3 LPOs (Davis et al., 2013). The trajectories constructed in that study required two impulsive burns: a tangential burn executed to depart from LEO, and a halo orbit injection maneuver upon arrival. The injection maneuver was always performed in the direction of the velocity component of the halo orbit's stable eigenvector, propagated to the injection location. The total ΔVs for the transfers ranged between 3.14 km/s and 3.24 km/s and the time of flight (TOF) varied from 44 to 90 days. The orbit insertion maneuvers ranged from 106 m/s to 200 m/s. In this study, we extend upon our previous work in order to reduce the TOF and/or the insertion maneuver by allowing the direction of ΔV_2 to vary.

2. Background

The equations of motion for the Circular Restricted Three-Body Problem (CRTBP) have been used for trajectory propagation in this study. The CRTBP models the motion of a particle of negligible mass under the influence of two larger bodies, termed the primaries. Conventionally, the larger of the bodies is termed the primary and the smaller is termed the secondary. The primaries rotate in circular orbits about the system barycenter. Additionally, the reference frame rotates about the barycenter at the same rotation rate as the two primaries. The x-axis extends from the origin through the secondary, the z-axis extends in the direction of the angular momentum of the system, and the y-axis completes the right-hand coordinate frame.

When performing analysis in the CRTBP, it is useful to normalize the system so certain quantities are dimensionless. The system is normalized such that the sum of the masses, the distance between the primaries, and the gravitational parameter all equal one, and the orbital period is normalized to 2π. These values are normalized by a Three-Body parameter μ, which is defined as the ratio of the smaller primary's mass to the sum of the mass of the two primaries:

$$\mu = \frac{m_2}{m_1 + m_2}, \tag{1}$$

where m_1 and m_2 are the masses of the primaries and $m_1 > m_2$. After normalizing, the locations of the primary and secondary become $(-\mu, 0, 0)$ and $(1-\mu, 0, 0)$, respectively.

The equations describing the motion of the third body may be written as:

$$\begin{aligned} \ddot{x} &= 2\dot{y} + x - (1-\mu)\tfrac{x+\mu}{r_1^3} - \mu\left(\tfrac{x-1+\mu}{r_2^3}\right) \\ \ddot{y} &= -2\dot{x} + y - (1-\mu)\tfrac{y}{r_1^3} - \mu\tfrac{y}{r_2^3} \\ \ddot{z} &= -(1-\mu)\tfrac{z}{r_1^3} - \mu\tfrac{z}{r_2^3}, \end{aligned} \tag{2}$$

where r_1 and r_2 represent the distance from the third body to the primary and secondary, respectively:

$$r_1^2 = (x+\mu)^2 + y^2 + z^2 \tag{3}$$

$$r_2^2 = (x-1+\mu)^2 + y^2 + z^2. \tag{4}$$

The reader is directed to Szebehely (1967) for a derivation of the equations of motion. The equations derived by Szebehely differ slightly than the ones presented here, as the x-axis in Szebehely extends from the origin through the primary. The Jacobi Constant, C, is an integral of motion which emerges in this dimensionless, rotating system: