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Rayleigh–Bénard and schwarzschild instability in a supercritical fluid

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Abstract

Due to the density stratification, the convection onset criterion in a supercritical fluid layer heated from below includes a stabilizing effect known as the Schwarzschild criterion, which requires an adaptation of the classical low Mach number approximation. We numerically solve the mathematical model describing the motion of a supercritical fluid in the classical Rayleigh–Bénard configuration and we bring to the fore two main events: (i) the reverse transition of an unstable layer to stability without any external intervention; (ii) the convection onset according to the Schwarzschild criterion. In addition, the 3D extension of the study is briefly described.

Keywords: Supercritical fluid; Piston effect; Rayleigh–Bénard configuration; Schwarzschild criterion; 3D convective instability

1. Introduction

In the past 10 years, many works were devoted to the natural convection inside a fluid close to its gas–liquid critical point. The Rayleigh–Bénard configuration received a lot of attention within the context of interaction between convection and the piston effect, a thermoacoustic effect responsible for the fast temperature equilibrium in a near-supercritical fluid (NCF) (Nitsche and Straub, 1987; Boukari et al., 1990; Onuki et al., 1990; Zappoli et al., 1990). Experimental (Kogan et al., 1999; Kogan and Meyer, 2001) and numerical (Raspo et al., 1999; Amiroudine et al., 2001; Accary et al., 2004) studies confirmed the pioneering results of the theoretical analysis obtained in 1970s by Gitterman and Steinberg (1970). These theoretical results showed that, owing to the divergence of the isothermal compressibility of the NCF, the classical Rayleigh criterion should be modified to take into account the compressibility of the fluid with respect to the hydrostatic pressure, thus including a stabilizing contribution, the Schwarzschild criterion or adiabatic temperature gradient criterion, usually encountered in atmospheric sciences for large air columns. Moreover, they revealed that, very close to the critical point, the stability of a NCF layer is governed by this Schwarzschild criterion, while far from it, the classical Rayleigh criterion takes control. The crossover between the regimes dominated by each of these two criteria was observed experimentally by Kogan and Meyer (2001). On the other hand, owing to an adaptation of the classical low Mach number approximation (Accary et al., in press), the direct numerical simulations (DNS) confirmed the relevance of the stabilizing effect of the Schwarzschild contribution; indeed, the works (Raspo et al., 1999; Amiroudine et al., 2001) show among others the existence of a cut-off temperature gap inside the fluid layer under which no convective motion is triggered no matter how thick the layer gets.

In this paper, we numerically solve the Navier–Stokes equations which describe the motion of a supercritical fluid in a bidimensional approximation. We show:

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(i) that taking into account the density stratification in the model is fundamental for the prediction of the convective instability threshold; (ii) that the Schwarzschild stabilizing effect can lead to a reverse transition towards a stable state; (iii) how different paths may trigger convection according to either of the two criteria; (iv) some preliminary 3D results.

2. Model

A NCF is contained between two infinite and no-slip plates, spaced away by H (H = 15 mm), and subjected to the earth gravitational field g (Fig. 1). Initially, the fluid is in thermodynamic equilibrium at a constant temperature T_i slightly above the critical temperature T_c such that $T_i = (1 + \varepsilon)T_c$, where ε defines the dimensionless proximity to the critical point ($\varepsilon \ll 1$). Under the effect of its weight, the fluid is stratified in density and pressure: with a mean density equal to its critical value ρ_c. The DNS starts by increasing the bottom plate temperature by ΔT (~few mK) while maintaining the top one at its initial temperature T_i.

The mathematical model for a NCF flow is described by the Navier–Stokes and energy equations written for a Newtonian and highly conducting van der Waals fluid (Zappoli et al., 1990). Despite of its simplicity, the van der Waals equation of state: yields to a critical divergence as ε^{-1} of the thermal expansion coefficient, of the isothermal compressibility, and of the heat capacity at constant pressure. $\Lambda(1 + \Lambda(T/T_c - 1)^{1/2})$ law, with $\Lambda = 0.75$ is used to describe the critical divergence of the thermal conductivity, while the heat capacity at constant volume and the dynamic viscosity are those of a perfect gas. We consider the carbon dioxide critical coordinates ($T_c = 304.13$ K, $\rho_c = 467.8$ Kg/m^3) and transport properties. The simulations were carried out for $T_i - T_c = 1$ K.

3. Numerical method

The governing equations are solved using a fully implicit finite-volume method on a staggered mesh. The method is second order accurate in space and third order in time and has been thoroughly validated on an analytical solution and on several benchmark tests of

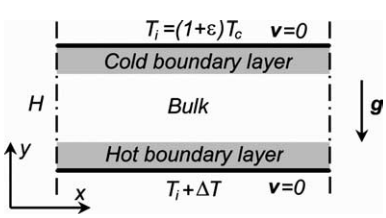

Fig. 1. A NCF in the Rayleigh–Bénard configuration.

natural convection (Accary and Raspo, accepted). The computational domain is a rectangle (height H: 15 mm, width: 10 mm) and periodic conditions are considered in the x-direction to simulate infinite horizontal extend. The mesh size for a grid-independent solution depends on the heating applied to the bottom plate, the finest being of (140 × 160) computational points. For an accurate description of the solution in the boundary layers, the mesh is refined with a power law near the bottom and top plates while it is uniform in the horizontal direction.

4. Acoustic filtering

4.1. Adapted low Mach number approximation

In the classical acoustic filtering, the pressure field is splitted into two parts: a dynamic pressure only involved in the momentum equations, and a space homogeneous time-dependent part that only appears in the energy and state equations. Owing to the divergence of the isothermal compressibility of the NCF, the hydrostatic pressure induces a density variation comparable to that resulting from the weak heating that we consider. Therefore, the compressibility of the NCF with respect to the hydrostatic pressure should be taken into account; therefore, an adapted low Mach number (ALMN) approximation was set up. Like the classical low Mach number (CLMN) approximation, in the adapted one, pressure is splitted into a dynamic part and a time-dependent part which includes now the density stratification. This ALMN approximation differs from the classical one in which the fluid compressibility is completely ignored. However, the assumption of small heating induces some simplifications and the resulting model requires no further numerical effort. In the Rayleigh–Bénard configuration, the proposed model allows taking account of the stabilizing effect of the hydrostatic pressure and predicts the convection threshold according to the criterion obtained by means of analytical analysis (Accary et al., in press).

4.2. Comparison with the classical low Mach number approximation

Comparisons between the classical model (CLMN) and the adapted one (ALMN) were carried out for several heating cases.

A brief description of the flow is appropriate before proceeding to the comparisons. The bottom heating induces a thin hot boundary layer (HBL) in which the density shows large variation due to the divergence of the thermal expansion coefficient of the NCF. This hot layer expands upward compressing adiabatically the rest of the fluid and leading to a fast increase of the temper-