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Influence of an inner core on the long-period forced librations of Mercury

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ABSTRACT

The planetary perturbations on Mercury's orbit lead to long-period forced librations of Mercury's mantle. These librations have previously been studied for a planet with two layers: a mantle and a liquid core. Here, we calculate how the presence of a solid inner core in the liquid outer core influences the long-period forced librations. Mantle–inner core coupling affects the long-period libration dynamics mainly by changing the free libration: first, it lengthens the period of the free libration of the mantle, and second, it adds a second free libration, closely related to the free gravitational oscillation between the mantle and inner core. The two free librations have periods between 2.5 and 18y depending on the internal structure. We show that large amplitude long-period librations of a few tens of arcsec are generated when the period of a planetary forcing approaches one of the two free libration periods. These amplitudes are sufficiently large to be detectable by spacecraft measurements of the libration of Mercury. The amplitudes of the angular velocity of Mercury's mantle at planetary forcing periods are also amplified by the resonances, but remain much smaller than the current precision of Earth-based radar observations unless the period is very close to a free libration period. The inclusion of mantle–inner core coupling in the rotation model does not significantly improve the fit to the radar observations. This implies that it is not yet possible to determine the size of the inner core of Mercury on the basis of available observations of Mercury's rotation rate. Future observations of the long-period librations may be used to constrain the interior structure of Mercury, including the size of its inner core.

1. Introduction

Mercury has a peculiar rotation: three rotation periods correspond to two revolution periods. This spin–orbit resonance leads to interesting physical phenomena such as the longitudinal librations. The librations are caused by the non-spherical mass distribution of Mercury, on which the Sun exerts a gravitational torque. The difference between the orbital and the rotation periods leads to a varying torque along the orbit since the orientation of the long axis of Mercury changes with respect to the direction to the Sun. Eccentricity further contributes to the variability by causing changes in the distance to the Sun and in the orbital speed. The main libration has a period of 87.97 days, equal to Mercury's orbital (annual) period, and an amplitude of 38.5 ± 1.6 arcsec (Margot et al., 2012). In addition, there are smaller amplitude librations at harmonic (semi-annual, ter-annual, etc.) frequencies. Those librations depend on the interior structure, notably the presence and the size of a liquid core inside the planet. By measuring the librations, we can infer knowledge about the interior structure. For example, by measuring Mercury's 88d libration amplitude, Margot et al. (2007) concluded that Mercury has a large liquid core. Since a magnetic field has been detected by the Mariner 10 spacecraft (e.g. Ness et al., 1975), it is thought that Mercury may have a solid inner core inside its liquid core. Peale et al. (2002) and Veasey and Dumberry (2011) have investigated the consequence of the addition of a solid inner core on the rotation dynamics of Mercury. Recently, Van Hoolst et al. (2012) showed that, if the inner core is larger than about 1000 km, the difference on the 88d libration amplitude may be non-negligible, and of the same order as the present uncertainty, about 1.5 as.

Another forced libration results from planetary perturbations. The periodic force arising from the gravitational interaction of a planetary body with Mercury causes a perturbation of Mercury's orbital motion, changing its position relative to the Sun and thus altering the solar torque acting on its equatorial bulge. This is an indirect effect of the planets on the rotation of Mercury. These long-period forced librations induced by the planetary perturbations have been predicted by e.g. Dufey et al. (2008), Peale et al. (2009) and Yseboodt et al. (2010). They have periods commensurate with the orbital revolution of the planets involved and are

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expected to have small amplitudes unless their period is close to the period of a free libration in which case a near-resonant amplification can occur. In the absence of an inner core, there is only one such mode, the free mantle libration. This mode describes an oscillation of the axis of minimum moment of inertia about the Mercury–Sun line at perihelion (Peale, 2005). The period of the free mantle libration depends on the moments of inertia of Mercury, and is approximately 12 years. This is very close to Jupiter's perturbation on Mercury's orbit at 11.86y; Dufey et al. (2008) and Peale et al. (2009) have shown that a forced libration of 20 arcsec or more can be generated, the exact amplitude depending on the moments of inertia. Besides this 11.86y forced libration, at least four other long-period forced librations have amplitudes larger than the arcsecond level.

The previous studies on the long-period forced librations assumed no mantle–inner core coupling. Adding an inner core has two effects on the free libration: First, as shown by Peale et al. (2002), Veasey and Dumberry (2011) and Van Hoolst et al. (2012), it can lengthen the period of the free libration of the mantle since the motion of the mantle is locked to that of the inner core for this mode. As a result, interior models with a large inner core may no longer have a free period close to Jupiter's orbital period. Second, the presence of the inner core adds a second free libration, closely approximated by the free gravitational oscillation between the mantle and inner core, and thus the possibility of additional resonances at other orbit perturbation frequencies.

In this study, we investigate how a non-spherical inner core coupled to the mantle and the outer core may influence the long-period forced librations. Since planetary perturbation periods may be close to the period of the two free modes, the long-period librations may be resonantly enhanced and this may lead to a rotation angle that substantially differs from a model where there is no solid inner core. The presence of the solid inner core affects the rotation state of Mercury and may result in a signature that is detectable in the observations, in which case it must be taken into account when analyzing the data. In our rotation model, we also take into account the dissipation since this effect reduces the libration amplitudes and introduces phase lags. We include viscous and electromagnetic coupling at the core–mantle and inner–outer core boundaries, as well as the effect of viscous deformation within the inner core. The signature of the parameters responsible for the dissipation on the libration is discussed.

In the theory section, we derive equations for the amplitude of the long-period forced librations of the mantle and of the inner core. The equations are given for cases with and without dissipation. We then numerically evaluate these libration equations on the basis of recent interior models of Mercury's (Rivoldini et al., 2009) (Section 1.7). The results are compared for different interior models. In the results section (Section 2), we show that the amplitude of the long-period librations are of the order of a few arcsec, and much larger if the forcing period approaches the period of one of the free modes. In the last section (Section 3), we compare predictions of libration models with and without mantle–inner core coupling with the Earth-based radar observations of the rotation rate of Mercury of Margot et al. (2012) in order to determine whether the size of the inner core can be determined on the basis of the currently available radar data.

2. Theory

2.1. Equations of motion for the mantle and the solid inner core

We assume an equatorial flattened bi-axial model of Mercury with a silicate shell composed of the mantle and the crust (we use the symbol m for the shell), a fluid outer core (oc) and the solid inner part of the core (ic). If the mantle and the inner core have a different rotation, their principal axes of inertia will be misaligned and there will be an effect of the gravitational and pressure coupling between these layers (see for example Van Hoolst et al. (2012)). The librational motion of the mantle and the solid inner core can be described by considering the change in angular momentum of these layers as a result of the external torque of the Sun and the internal torques. For the libration equation, we assume that the mantle and inner core are rigid solids as the effect of elastic deformations has been shown to be below the observational detection limit (Van Hoolst et al., 2012). We then have the equations of motion:

$$\ddot{\psi}_m = -\frac{GM_S}{C_m r^3 n^2} K_m \sin 2(\psi_m - \varpi - f) - \frac{K}{C_m} \sin 2(\psi_m - \psi_{ic}), \quad (1)$$

$$\ddot{\psi}_{ic} = -\frac{GM_S}{C_{ic} r^3 n^2} K_{ic} \sin 2(\psi_{ic} - \varpi - f) + \frac{K}{C_{ic}} \sin 2(\psi_m - \psi_{ic}). \quad (2)$$

The rotation angle of the mantle ψ_m describes the orientation of the axis of minimum moment of inertia of the mantle A_m relative to the intersection between the ecliptic and the orbital plane at J2000. Similarly, ψ_{ic} is the rotation angle of the inner core. f is the true anomaly, ϖ the longitude of the pericenter, r the distance between the mass centers of Mercury and the Sun, $A_{ic} < B_{ic} < C_{ic}$ are the principal moments of inertia of the inner core and $A_m < B_m < C_m$ the mantle moments of inertia. n is the mean motion of Mercury, M_S the mass of the Sun and G the gravitational constant. The factors K_m and K_{ic} describe the strengths of the gravitational and pressure torques on the mantle and inner core, respectively (Van Hoolst et al., 2012; Dumberry et al., 2013). These factors are defined by $K_m = \frac{3}{2} n^2 (B_m - A_m + B_{oc,t} - A_{oc,t})$ and $K_{ic} = \frac{3}{2} n^2 (B_{ic} - A_{ic} + B_{oc,b} - A_{oc,b})$, where $A_{oc,b}$ and $B_{oc,b}$ are the principal moments of inertia of the bottom part of the fluid core (a layer between the inner core–outer core boundary (ICB) and the smallest sphere that can be included in the fluid core, see Fig. 2 of Van Hoolst et al. (2008)) while $A_{oc,t}$ and $B_{oc,t}$ are related to the rest of the fluid core. The terms proportional to $B_m - A_m$ and $B_{ic} - A_{ic}$ capture the solar gravitational torques on the mantle and inner core, respectively. The additional terms arise from the pressure torques on the boundaries between the outer core and mantle and between the inner core and outer core. It can be shown that in the limit of no inner core, the expression of K_m reduces to (3/2) $n^2(B - A)$, and we retrieve the classical equation of a planet with two layers. K is the gravitational-pressure coupling constant between the mantle and the inner core. If the inner and outer parts of the core have uniform density ρ_j ($j = oc$ for the fluid outer core, $j = ic$ for the solid inner core and $j = m$ for the silicate shell), K is defined by (e.g. Veasey and Dumberry, 2011)

$$K = \frac{4\pi G}{5} \left(1 - \frac{\rho_{oc}}{\rho_{ic}}\right) C_{ic} \beta_{icb} [(\rho_{oc} - \rho_m) \beta_{cmb} + \rho_m \beta_m], \quad (3)$$

where β_m, β_{cmb} and β_{icb} are the geometrical equatorial flattenings at the top of the mantle, core–mantle boundary (CMB) and ICB, respectively. If an ellipsoidal surface of constant density at a given radius has its three principal semi-axes defined by $a > b > c$, the geometrical flattening in the equatorial plane is defined by $\beta = (a - b)/a$. For the computation of the longitudinal librations, the polar flattening may be neglected. When radial density variations in both the fluid and solid cores are taken into account, the expression for K is more complicated and given in Dumberry et al. (2013). Since the effect of the small obliquity on the longitudinal librations is below the observational detection limit, the obliquity of Mercury is assumed to be 0 (its true value is 0.034°, Margot et al., 2012) so that Mercury's equator and orbit are the same plane.

Previous studies of the effect of the inner core on Mercury's rotation focused on the amplitude of the 88d librations and considered a Kepler orbit, in which the orbital elements are constant with