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Insolation on exoplanets with eccentricity and obliquity

Anthony R. Dobrovolskis *

245-3 NASA Ames Research Center, Moffett Field, CA 94035-1000, United States

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ABSTRACT

The pattern of insolation on an extrasolar planet has profound implications for its climate and habitability. A planet's insolation regime depends on its orbital eccentricity, the obliquity of its spin axis, its rotation rate, and its longitude of vernal equinox. For example, although a planet receives the same time-averaged insolation at both poles, the *peak* insolation at its poles can differ by a factor up to 27, depending on its eccentricity and equinox. This is of particular interest for planets with polar icecaps (or lakes and seas), like Mercury, Earth, and Mars (or Titan). The nearly 600 exoplanets now with known eccentricities span a wide range of eccentricity from essentially zero up to near unity; but their obliquities are still unknown, and also may range widely.

Including both non-zero eccentricity and obliquity together vastly broadens the variety of global insolation patterns on extrasolar planets. This applies especially to planets in synchronous rotation, or in other spin–orbit resonances (like Mercury), which can exhibit quite complicated and unusual insolation patterns. For example, regions of eternal daylight and endless night occur only on synchronous exoplanets, whose rotation periods equal their orbital periods; but the peak and time-averaged insolation can vary by factors of at least 32 and 88, respectively, over a planet with a rotation period of half its orbital period, an eccentricity of 0.20, and an obliquity of 60°.

Patterns of both mean and peak insolation display various symmetries with respect to latitude and longitude on the planet's surface. Most of these are relatively simple and easily understood; for example, a resonant planet whose orbital period is half of an odd multiple of its rotation period (as in Mercury's 3/2 resonance) experiences identical insolation patterns at longitudes 180° apart. However, such half-odd resonances also exhibit a totally unexpected symmetry of the time-averaged insolation with respect to the planet's equator, not shared by the peak insolation, or by any whole-number resonances. This emergent symmetry can be understood by Fourier analysis of the time-varying insolation.

1. Introduction

One of the greatest surprises concerning extrasolar planets (also known as exoplanets) has been their wide range of orbital eccentricities, compared to the planets of our Solar System. The latest version of the exoplanet orbit database (Wright et al., 2011, updated to June 2012) now lists 587 verified exoplanets with reported eccentricities e ranging from zero up to 0.934! Of these, 163 (or ~28%) are tabulated simply as 0, while 202 (or ~34%) have $e \geqslant 0.20$, and 50 (or ~8.5%) have $e \geqslant 0.50$. The median eccentricity is ~0.110, while the first and third quartiles are 0 and 0.266. The mean and Root-Mean-Square values of e are ~0.172 and 0.415, respectively, implying a standard deviation of $\sqrt{0.415^2 - 0.172^2} \approx 0.377$.

Fig. 1 of Dobrovolskis, 2007, hereinafter referred to as Paper 1) plotted the 209 eccentricities then listed in the previous exoplanet catalog (Butler et al., 2006, updated to May 2007) versus their orbital semi-major axes a, to demonstrate the gradual fall-off of the range of e from near unity to near zero as a decreases from ~0.5 Astronomical Units down to ~0.01 AU. This fall-off has been widely attributed to damping of eccentricity by dissipation of tides raised in close-in planets by their parent stars.

However, this is somewhat misleading insofar as it suggests that a remains constant while e decays. In fact, dissipation of tidal energy in exoplanets decreases both a and e, but such that the planet's orbital angular momentum is conserved, and its "semi-latus rectum" $a[1 - e^2]$ remains constant. Fig. 2 of Paper 1 showed that a plot of e versus $a[1 - e^2]$ has a much steeper cut-off of the range of e around 0.05 AU. This implies that semi-latus rectum is more fundamental than semi-major axis for tidal damping.

Repeating both of these plots with the latest data (Wright et al., 2011) shows little qualitative change from Figs. 1 and 2 of Paper 1, although the number of exoplanets with known eccentricities has more than doubled since then, from 209 to 587, as reported on the exoplanet orbit database (Butler et al., 2006; Wright et al., 2011). Furthermore, the eccentricities are not obviously related with the

* Fax: +1 650 604 6779.
E-mail address: anthony.r.dobrovolskis@nasa.gov

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Nomenclature

a	orbital semi-major axis
e	orbital eccentricity
f,g	arbitrary functions
$\dot{f}$	time derivative: $\dot{f} \equiv df/dt$
$\underline{f}$	half-rectified function: $\underline{f} = f/2 + \lvert f/2 \rvert$
F	unrectified insolation: $F \equiv S_0 \cos\zeta$
h	$h \equiv e\sin\alpha$
k	$k \equiv e\cos\alpha$
L^*	stellar luminosity
n	orbital mean motion: $n \equiv 2\pi/P$
p	$p \equiv \omega/n$
P	orbital period
S	incident flux at surface: $S = \underline{F}$
S_*	extra-solar constant: $S_* \equiv L^*/(4\pi a^2)$
S_0	incident flux at sub-solar point
$\bar{S}$	time-averaged insolation
$\widehat{S}$	maximum incident flux
$\check{S}$	minimum incident flux
\$	$\$ \equiv \underline{-F} = \lvert F/2 \rvert - F/2$
t	time since periapsis
x,y,z	Cartesian coordinates in the planet-fixed frame
X,Y,Z	Cartesian coordinates in the non-rotating frame
X_*,Y_*,Z_*	inertial coordinates of sub-solar point
α	argument of periapsis, or longitude of vernal equinox
β	axial obliquity
γ	hour angle from vernal equinox to planet's prime meridian
ζ	angle of star from local zenith
λ	latitude on planet's surface
Λ	latitude in inertial frame
Λ_*	inertial latitude (or declination) of sub-solar point
Λ_0	inertial latitude of periapsis
ν	true anomaly along orbit
ϕ	longitude on planet's surface
Φ	longitude in inertial frame
Φ_*	inertial longitude of sub-solar point
Φ_0	inertial longitude of periapsis
ω	rotation rate of planet

angle between periapsis and the line of sight, which would be evidence of systematic error.

The eccentricities of exoplanets are significant because it has often been asserted that a mostly solid planet (like Earth, or a "super-Earth") close to its parent star must be in synchronous rotation, so that its rotation period equals its orbital period, and it always keeps nearly the same hemisphere toward its star, just as our Moon always keeps nearly the same face toward the Earth. However, this synchronous resonance applies primarily to planets in nearly circular orbits ($e \lesssim 0.2$); a close-in planet with higher eccentricity is liable to be found in a higher spin–orbit resonance, such that its orbital period is an integer multiple of half its rotation period (see Paper 1). For example, in our own Solar System, the planet Mercury ($e \approx 0.206$) rotates exactly three times during every two orbits.

It is clear that eccentricity and rotation rate exert a profound influence on a planet's climate and habitability, through the distribution of insolation over its surface. For example, a synchronous exoplanet in a nearly circular orbit, keeping one hemisphere always turned toward its sun while the other always remains

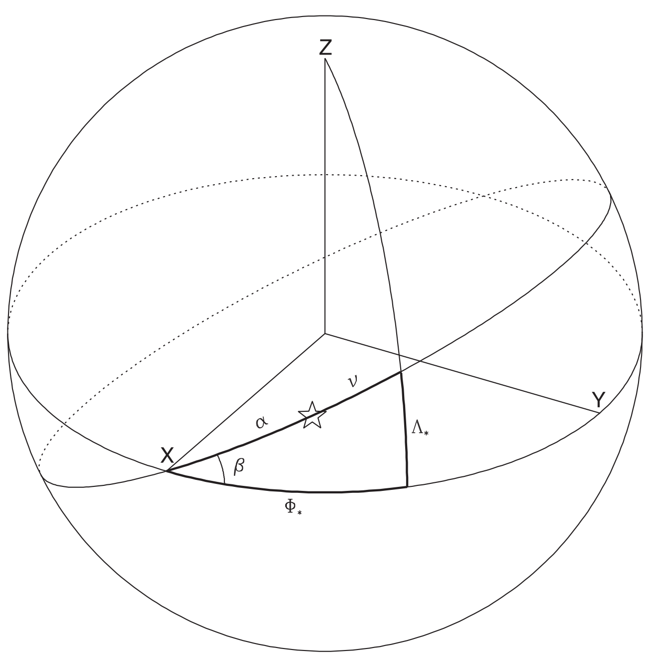

Fig. 1. Coordinates used in text (non-rotating). The large circle delimits the celestial sphere. The horizontal oval represents the planet's equatorial plane, while the tilted oval depicts its orbital plane, inclined to the equator by an angle β, as shown. The Z-axis lies along the planet's right-hand rotational pole, while the X-axis lies along its vernal equinox (the ascending node of the star on the planet's equator), and the Y-axis completes the right-handed XYZ triad. The sub-solar point at time t after vernal equinox lies at inertial longitude Φ_*, latitude Λ_*, and (signed) angular distance $\alpha + \nu(t)$ from the X-axis, as indicated by the heavy right triangle. The pentagram represents the sub-solar point at periapsis.

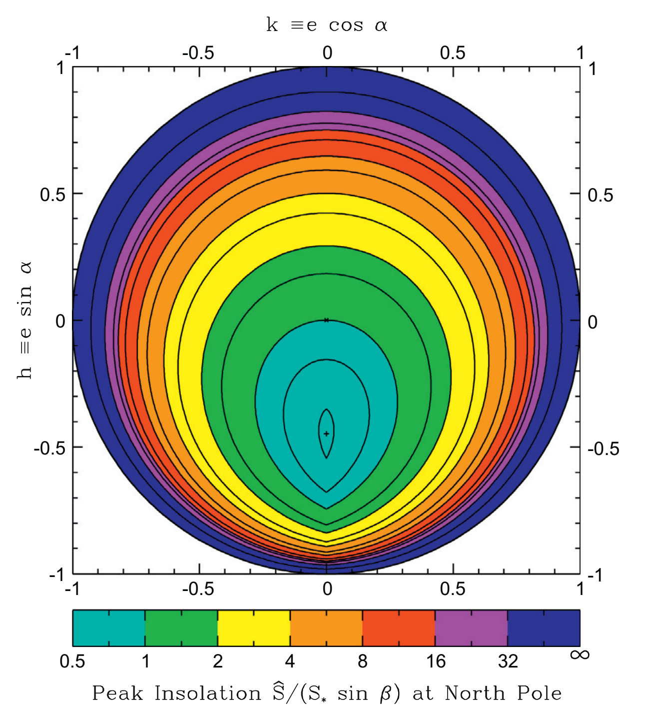

Fig. 2. Peak insolation $\widehat{S}_N$ at the north pole (normalized by $S_* \sin\beta$), as a function of polar coordinates e and α, or of rectangular coordinates h and k. Contours: 0.55, 0.75, 1.0, 1.5, 2, 3, 4, 6, 8, 12, 16, 20, 32, 100, and ∞. The $\times$ through the unit contour denotes the origin $e = 0$, while the + denotes the minimum of $\widehat{S}_N$.