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Nonhydrostatic effects and the determination of icy satellites' moment of inertia

Peter Gao *, David J. Stevenson

Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125, USA

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ABSTRACT

We compare the moment of inertia (MOI) of a simple hydrostatic, two layer body as determined by the Radau–Darwin Approximation (RDA) to its exact hydrostatic MOI calculated to first order in the parameter $q = \Omega^2R^3/GM$, where Ω, R, and M are the spin angular velocity, radius, and mass of the body, and G is the gravitational constant. We show that the RDA is in error by less than 1% for many configurations of core sizes and layer densities congruent with those of solid bodies in the Solar System. We then determine the error in the MOI of icy satellites calculated with the RDA due to nonhydrostatic effects by using a simple model in which the core and outer shell have slight degree 2 distortions away from their expected hydrostatic shapes. Since the hydrostatic shape has an associated stress of order $\rho\Omega^2R^2$ (where ρ is density) it follows that the importance of nonhydrostatic effects scales with the dimensionless number $\sigma/\rho\Omega^2R^2$, where σ is the nonhydrostatic stress. This highlights the likely importance of this error for slowly rotating bodies (e.g., Titan and Callisto) and small bodies (e.g., Saturn moons other than Titan). We apply this model to Titan, Callisto, and Enceladus and find that the RDA-derived MOI can be 10% greater than the actual MOI for nonhydrostatic stresses as small as ∼0.1 bars at the surface or ∼1 bar at the core–mantle boundary, but the actual nonhydrostatic stresses for a given shape change depends on the specifics of the interior model. When we apply this model to Ganymede we find that the stresses necessary to produce the same MOI errors as on Titan, Callisto, and Enceladus are an order of magnitude greater due to its faster rotation, so Ganymede may be the only instance where RDA is reliable. We argue that if satellites can reorient to the lowest energy state then RDA will always give an overestimate of the true MOI. Observations have shown that small nonhydrostatic gravity anomalies exist on Ganymede and Titan, at least at degree 3 and presumably higher. If these anomalies are indicative of the nonhydrostatic anomalies at degree 2 then these imply only a small correction to the MOI, even for Titan, but it is possible that the physical origin of nonhydrostatic degree 2 effects is different from the higher order terms. We conclude that nonhydrostatic effects could be present to an extent that allows Callisto and Titan to be fully differentiated.

1. Introduction

Current models of icy satellite formation and evolution depend on the accuracy with which we determine their interior structures. These can be inferred from their moments of inertia (MOI), which can be estimated from *in situ* gravitational field measurements by spacecraft. The primary method of estimation is the Radau–Darwin Approximation (RDA) (e.g., Hubbard, 1984; Murray and Dermott, 1999), which relates the MOI to the degree 2 response of the body to rotation and tides expressed in the gravitational coefficients J_2 and C_{22}, defined by:

$$J_2 = \frac{C}{MR^2} - \frac{B+A}{2MR^2} \quad (1a)$$

$$C_{22} = \frac{B-A}{4MR^2} \quad (1b)$$

where M and R are the mass and mean radius of the body, respectively; and $C > B > A$ are the principle moments of inertia of the body. The accuracy of RDA for a two layer body has been explored in works such as Zharkov (2004) and Schubert et al. (2011), where the results of RDA are compared to the exact solution of Kong et al. (2010) and the results of the theory of figures, yielding relatively small differences of ∼0.1%. The RDA makes three assumptions: (1) The body is in hydrostatic equilibrium; (2) there are no large density variations; (3) the perturbations arising from tides and rotation are small (i.e., linear response). Our primary focus here is on the first assumption. We note, however, that assumption (2) seems to have been insufficiently explored in the published literature and we accordingly have included brief consideration of this approximation here. Assumption (3) is violated for gas and ice giants

* Corresponding author.
E-mail address: pgao@caltech.edu (P. Gao).

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because of their rapid rotation and this is the focus of the aforementioned theory of figures discussed in Hubbard (1984) and revisited very recently in Hubbard (2012) in the context of Maclaurin spheroids, the same approach that we use (in the linear limit).

The RDA has been used to determine the MOIs of several large icy satellites, such as Titan (Iess et al., 2010), Ganymede (Anderson et al., 1996), and Callisto (Anderson et al., 2001), as well as medium-sized satellites, such as Rhea (Iess et al., 2007). Enceladus is of great interest but no consensus has emerged yet on the MOI for this body. Our main focus here is on the large icy bodies for which it is commonly assumed that RDA is accurate, but we will also briefly discuss the impact of RDA errors on Enceladus in anticipation of future results. Table 1 shows several physical and orbital parameters for these bodies, including their MOIs determined from RDA if available. Despite the similarities in the masses and radii of Titan, Ganymede, and Callisto, their determined moments of inertia appear to be vastly different: Ganymede's is low, implying full differentiation, while Titan and Callisto's are high, implying partial differentiation. This interpretation has influenced models for satellite formation such as the "gas-starved disk" model for the Galilean moons (Stevenson, 2001; Canup and Ward, 2002), and alternative ways of reducing accretion heating through lengthening the timescale of accretion (e.g. Mosqueira and Estrada, 2003). These models may avoid differentiation during accretion but differentiation may occur subsequently. In fact, the full differentiation of Ganymede is often attributed to later processes in this model such as tidal heating (Canup and Ward, 2002), though differences in the formation environment such as a higher disk temperatures at Ganymede's orbital distance (Barr and Canup, 2008) are also proposed. Some models avoid the full differentiation of Callisto and Titan due to later processes by constraining the formation times so as to avoid excessive heating by short-lived radioisotopes (Barr et al., 2010), while others allow for full differentiation of Titan but with a low-density, hydrated silicate core (Fortes, 2012; Castillo-Rogez and Lunine, 2010). Still others maintain that Callisto and Titan must fully differentiate in the lifetime of the Solar System due to density gradients trapping heat generated by long-lived radioisotopes (O'Rourke and Stevenson, 2013).

Ultimately, the usefulness of these models depends on the accuracy of the MOIs that they attempt to explain, which in turn depends on the reliability of the RDA in determining the MOIs from the gravity measurements. The RDA predicts a one-to-one correspondence between MOI and J_2 for a specific rotation, but nonhydrostatic effects destroy this correspondence by introducing a nonhydrostatic contribution to J_2 that cannot be easily separated out from the measured J_2 value. This could be especially troublesome in slowly rotating bodies where the nonhydrostatic contribution could make up a large fraction of the total J_2, whereas the effect would be less in fast rotating bodies. For instance, Mueller and McKinnon (1988) noted that nonhydrostatic effects could be present on the slow rotator Callisto in magnitudes that would render its determined MOI untrustworthy. In this paper, we generalize this analysis to any degree 2 nonhydrostatic contribution and extend it to Titan and Ganymede, where long wavelength mass anomalies have been detected (Palguta et al., 2006; Iess et al., 2010). This will allow us to both evaluate the accuracy of Titan and Callisto's determined MOIs given nonhydrostatic effects and determine whether Ganymede is less affected by these effects given its faster rotation. As previously mentioned, we will also extend our analysis to Enceladus due to its unique nature. Considering that a 10% error in the MOI of Callisto and a few% for Titan could potentially result in values consistent with fully differentiated bodies, it is essential that the effects of these nonhydrostatic structures be determined to establish their impact on the calculated MOIs of the large icy satellites, and in turn our understanding of their interiors and evolutionary processes.

In Section 2, we first establish the error in RDA assuming exact hydrostatic equilibrium but allowing for large density differences in the context of a nested Maclaurin spheroid model. We then quantify the effects degree 2 nonhydrostatic anomalies have on the MOI of a generalized large icy satellite as determined by the RDA, as well as the relationship between the magnitude of the nonhydrostatic anomaly and the stress caused by such an anomaly on the icy satellite. In Section 3 we apply our model to Titan, Ganymede, Callisto, and Enceladus and assess whether existing nonhydrostatic contributions and/or other possible sources are capable of producing major MOI errors. Finally, we summarize our work and state our conclusions in Section 4.

2. Theory

2.1. Hydrostatic icy satellite model

The RDA can be expressed as

$$\frac{C}{MR^2} = \frac{2}{3}\left(1 - \frac{2}{5}\sqrt{\frac{5}{3\Lambda_{2,0}+1} - 1}\right) \tag{2}$$

where C/MR^2 is the nondimensionalized polar moment of inertia (Hubbard, 1984). For a body deformed only by rotation $\Lambda_{2,0} = J_2/q$ in the limit of small values of q, and

$$q = \frac{\Omega^2 R^3}{GM} \tag{3}$$

is a dimensionless measure of the "centrifugal potential" that arise due to rotation, with Ω, R, and M as the spin angular velocity, mean radius, and mass of the body, and G as the gravitational constant. For synchronously rotating satellites, the tidal potential is three

Table 1
Selected physical and orbital parameters of Titan, Ganymede, Callisto, and Enceladus.

	Titan	Ganymede	Callisto	Enceladus
Mass (10^{26} g)	1.3452 ± 0.0002[c]	1.48167 ± 0.0002[d]	1.0759 ± 0.0001[e]	0.00108 ± 1e−6[c]
Mean radius (km)	2574.73 ± 0.09[f]	2634.1 ± 0.3[g]	2408.4 ± 0.3[g]	252.1 ± 0.1[c]
Orbital period (Earth days)[a]	15.95[h]	7.15[h]	16.69[h]	1.37[h]
MOI[b]	0.3414 ± 0.0005[f]	0.3105 ± 0.0028[d]	0.3549 ± 0.0042[e]	Unknown
q (10^{-5})[i]	3.9545	19.131	3.6958	626.374

[a] Assumed to be the same as rotation period, i.e. synchronous rotation.
[b] Moment of inertia in units of C/MR^2, where C, M, and R are the polar moment of inertia, mass, and mean radius of the body in question.
[c] Jacobson et al. (2006).
[d] Anderson et al. (1996).
[e] Anderson et al. (2001). Callisto mass and associated uncertainty calculated from dividing given GM value and GM uncertainty by given G value.
[f] Iess et al. (2010) (SOL1 flybys).
[g] Showman and Malhotra (1999).
[h] Murray and Dermott (1999) (no uncertainties were given).
[i] $q = \Omega^2R^3/GM$, where Ω, R, and M are the spin/orbital angular velocity, radius, and mass, respectively, of the satellite; and G is the gravitational constant.