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Gravitational signature of rotationally distorted Jupiter caused by deep zonal winds

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ABSTRACT

Both deep zonal winds, if they exist, and the basic rotational distortion of Jupiter contribute to its zonal gravity coefficients J_n for $n \geqslant 2$. In order to capture the gravitational signature of Jupiter that is caused solely by its deep zonal winds, one must take into account the full effect of rotational distortion by computing the coefficients J_n in non-spherical geometry. This represents a difficult and challenging problem because the widely-used spherical-harmonic-expansion method becomes no longer suitable. Based on the model of a polytropic Jupiter with index unity, we compute Jupiter's gravity coefficients J_2, J_4, $J_6, \ldots, J_{12}$ taking into account the full effect of rotational distortion of the gaseous planet using a finite element method. For the model of deep zonal winds on cylinders parallel to the rotation axis, we also compute the variation of the gravity coefficients ΔJ_2, ΔJ_4, $\Delta J_6, \ldots, \Delta J_{12}$ caused solely by the effect of the winds in non-spherical geometry. It is found that the effect of the zonal winds on lower-order coefficients is weak, $|\Delta J_n/J_n| < 1\%$, for $n = 2, 4, 6$, but it is substantial for the high-degree coefficients with $n \geqslant 8$.

1. Introduction

The Juno spacecraft, now on its way to Jupiter, will make high-precision measurements of the gravitational field of the giant planet (Hubbard, 1999; Bolton, 2005; Kaspi et al., 2010). Interpretation of these gravity measurements requires (i) an accurate description of Jupiter's gravitational field in its equilibrium under the balance of self-gravity, internal pressure and strong rotational effects for which the effect of rotational distortion can no longer be treated as a small perturbation on a spherically symmetric state and (ii) the possible effects of its alternating, cloud level zonal flows on its gravitational field that are caused solely by the zonal winds in non-spherical geometry. This represents a mathematically difficult and challenging problem since the simple spherical-harmonic-expansion method is no longer suitable.

The gravitational potential V_g in the exterior of a rotationally distorted axially symmetric Jupiter can be expanded in terms of the Legendre functions P_n,

$$V_g(r,\theta) = -\frac{GM_J}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\cos\theta)\right], \quad r \geqslant R_e, \tag{1}$$

where M_J is Jupiter's mass, n takes even integers, (r, θ, ϕ) are spherical polar coordinates with $\theta = 0$ being at the axis of rotation, J_2, J_4, $J_6, \ldots$, are the zonal gravitational coefficients, R_e is the equatorial radius of Jupiter, and G is the universal gravitational constant ($G = 6.67384 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$). At present, only the first 3 zonal gravitational coefficients J_2, J_4, J_6 (see Table 1) are accurately measured (Jacobson, Jup230 orbit solution, http://ssd.jpl.nasa.gov/gravity_fields_op). By circling Jupiter in a polar orbit, the Juno spacecraft will carry out high-precision measurements of the gravitational coefficients up to J_{12} (Bolton, 2005). The primary objectives of this paper are twofold: (i) to obtain the zonal gravitational coefficients up to J_{12} for a polytropic Jupiter without treating the effect of rotational distortion as a small perturbation and (ii) to compute an upper bound of the zonal wind effect on the gravitational coefficients of rotationally distorted Jupiter with non-spherical geometry.

Despite the fact that Jupiter's zonal winds have been measured and studied for a number of decades, their generation and maintenance still remains highly controversial. There exist at least three different views of the origin and the depth of the zonal winds. One opinion is that they are generated by thermal convection within the deep interior of the planet, representing the azimuthal fluid motion that is nearly independent of distance parallel to the rotation axis and extending through the whole interior (Busse, 1976). A second opinion is that the winds are powered by internal heat via thermal convection, but they are only confined within the planet's outermost thin shell (about 10% of Jupiter's radius)

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Table 1
The second column gives the observed values, the third column is from the classical perturbation theory of Hubbard (1974) and the fourth to eighth columns show numerical convergence of the zonal gravity coefficients J_n up to $n = 12$. Here kM denotes $k \times 10^6$ tetrahedral elements in the spheroidal domain of our numerical computation. Recent computation by Hubbard (2013) using 512 concentric Maclaurin spheroidal layers gives $J_2 = 13989 \times 10^{-6}$, $J_4 = -531.87 \times 10^{-6}$ and $J_6 = 30.12 \times 10^{-6}$.

	Observation	Theory	2M	4M	8M	16M	32M
J_2	+14696.43	+13930	+14909.60	+14909.59	+14909.59	+14909.59	+14909.59
J_4	−587.14	−520	−559.07	−559.08	−559.08	−559.07	−559.08
J_6	+34.25	+39	+29.89	+29.90	+29.90	+29.90	+29.90
J_8			−1.817	−1.948	−1.950	−1.948	−1.949
J_{10}			+0.541	+0.140	+0.144	+0.143	+0.144
J_{12}			+0.236	−0.007	−0.012	−0.012	−0.011

(Heimpel et al., 2005) because the effect of magnetic braking and ohmic dissipation would limit their depth (Liu et al., 2008). The third view is that the alternating zonal flow is largely confined to a top thin layer of its stably stratified atmosphere (Ingersoll and Cuzzi, 1969; Williams, 1976; Lian and Showman, 2010; Liu and Schneider, 2010). Deep zonal winds of Jupiter, if they exist, can generate an external gravitational signature by inducing density anomalies (Hubbard, 1999) or by modifying the planet's shape (Kong et al., 2012).

By assuming that Jupiter is in hydrostatic equilibrium with a polytropic index of unity and that it rotates differentially on cylinders parallel to the rotation axis, Hubbard (1999) carried out the first study of the effect of deep zonal winds on Jupiter's gravity harmonics in spherical geometry. By assuming that Jupiter's zonal winds are driven by the thermal wind mechanism, Kaspi et al. (2010) solved the thermal–wind equation, based on the mean density model given by Guillot and Morel (1995), to determine the gravitational anomalies caused by the zonal winds, also in spherical geometry. It should be pointed out that there are mathematical limitations to the thermal–wind approach because the thermal wind equation is mathematically degenerate and its solution is known to be non-unique. The present study adopts the assumption used by Hubbard (1999) that the zonal winds extend through the whole interior of Jupiter, thus leading to an upper bound on the effect of the zonal winds on the gravitational coefficients.

Jupiter is rotating rapidly, resulting in significant departure from spherical geometry: its eccentricity at the one-bar surface is $\mathcal{E}_J = 0.3543$ (Seidelmann et al., 2007). Classical perturbation theories (Chandrasekhar, 1933; Zharkov and Trubitsyn, 1978) – which are based on an expansion around spherical geometry using a small rotation parameter – would find it difficult to reach the high precision (Bolton, 2005) anticipated in Juno's observations of Jupiter's gravitational field. As an alternative way of computing the gravitational field of a rotationally distorted gaseous body, Kong et al. (2013) developed a new three-dimensional numerical method for calculating the non-spherical shape and internal structure of a rapidly rotating gaseous body with a polytropic index of unity. Using a finite element mesh with 2×10^6 tetrahedral elements, they obtained the first three gravitational coefficients J_2, J_4, J_6 of Jupiter that take into account the full effect of its rotational distortion.

The objectives of the present study are twofold. By constructing a perturbation expansion taking into account the full rotational distortion, we investigate two problems related to Jupiter's gravitational coefficients. In the leading-order problem, we extend the previous study of Kong et al. (2013) by carrying out the accurate computation of the zonal gravitational coefficients J_2, J_4, J_6 up to J_{12} using a three-dimensional finite-element mesh in non-spherical geometry with 32×10^6 tetrahedral elements. In the next-order problem of the perturbation analysis, we compute the variations of the gravitational coefficients, $\Delta J_2, \Delta J_4, \ldots, \Delta J_{12}$ caused by the deep zonal winds on cylinders parallel to the rotation axis in non-spherical geometry. It is significant that the two problems are mathematically and physically coupled and inseparable. Mathematically, the leading-order problem determines the equation of state, the shape of a rotationally distorted Jupiter and the gravitational coefficient J_n based on which the next-order problem can compute its variation ΔJ_n caused by the zonal winds. Physically, gravity measurements by the Juno spacecraft provide only the total gravitational coefficients $J_n + \Delta J_n$. It follows that the values of accurate J_n are required in order to identify their wind-related variations ΔJ_n from the measured values $J_n + \Delta J_n$. In comparison to the approach based on the thermal wind equation in spherical geometry (Kaspi et al., 2010), the present approach is self-consistent and mathematically marked by the uniqueness of its solution, providing both accurate values of the gravitational coefficients J_n and their wind-related variations ΔJ_n in non-spherical geometry.

We begin by presenting the governing equations in Section 2. The numerical method is briefly discussed in Section 3 while the results are discussed in Section 4 and some remarks are given in Section 5.

2. Model and governing equations

Our model assumes that (i) Jupiter with mass M_J and equatorial radius R_e is isolated and rotating rapidly about the z-axis with an angular velocity $\Omega_0\hat{\boldsymbol{z}}$, (ii) Jupiter is axially symmetric and consists of a compressible barotropic fluid (a polytrope of index unity) whose density is a function only of pressure (Hubbard, 1999) and (iii) the zonal winds with a typical speed U observed on the outer surface of Jupiter represent the deep flows that depend only on distance s from the rotation axis and extend through the whole interior (Busse, 1976). In an inertial frame of reference, the fluid motion $\mathbf{u}$ of Jupiter is described by

$$\mathbf{u} = \Omega_0[1 + \epsilon\widehat{\Omega}(s)]\hat{\boldsymbol{z}} \times \mathbf{r}, \tag{2}$$

where $\epsilon = U/(R_e\Omega_0) \ll 1$, $\mathbf{r}$ is the position vector and $\epsilon\Omega_0\widehat{\Omega}(s)\hat{\boldsymbol{z}} \times \mathbf{r}$ represents the deep zonal winds which can be derived from the observed profile on Jupiter's surface by extension along cylinders parallel to the rotation axis (Hubbard, 1999; Kaspi et al., 2010). With a polytrope of index unity, the pressure (p)–density (ρ) relation in Jupiter is assumed to be

$$p = K\rho^2, \tag{3}$$

where K is a constant. In an inertial frame of reference, the equilibrium equations for a rotating gaseous Jupiter in an inviscid limit are

$$\mathbf{u} \cdot \nabla\mathbf{u} = -\frac{1}{\rho}\nabla p - \nabla V_g, \tag{4}$$

$$\nabla^2 V_g = 4\pi G\rho, \tag{5}$$

$$\nabla \cdot (\mathbf{u}\rho) = 0, \tag{6}$$

where $\mathbf{u}$ is given by (2) and V_g denotes the gravitational potential. Eqs. (3)–(6) must be solved subject to the two boundary conditions

$$p = 0 \text{ and } V_g + V_c = \text{constant at the bounding surface } \mathcal{S} \text{ of Jupiter}, \tag{7}$$