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Effects of Trojan exoplanets on the reflex motions of their parent stars

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ABSTRACT

This paper collects and corrects certain results on the radial velocity variations of stars in Trojan systems, where two exoplanets separated by 60° of longitude share the same orbit. Radial velocity measurements can seriously misjudge the masses and densities of planets in such systems.

I also derive a simple new method for combining tidal perturbations from two or more sources, such as the tides raised on Earth by the Sun and Moon. This method then shows that the observable effects of tides raised on stars by Trojan planets are also misleading. However, the combination of both tidal and radial velocity measurements can determine the mass of each planet. I conclude with a discussion of possible scenarios for the formation of co-orbital exoplanets.

1. Introduction

In our Solar System, the planets Jupiter, Neptune, Mars, and Earth share their orbits with certain asteroids, called "Trojans", which librate in "tadpole" orbits about the Lagrange equilibrium points L4 and L5, 60° ahead of and behind the planet, respectively. Saturn's classical moons Tethys and Dione each have two Trojan companions as well, while its co-orbital satellites Janus and Epimetheus describe mutual "horse-shoe" orbits around Saturn, which librate about both L4 and L5 with large amplitude (see Murray and Dermott, 1999).

Caton et al. (1999) (see also Davis et al., 2001) may have been the first to consider the possibility of planets at the Trojan points of a binary star system, and searched for transits of such exoplanets in eclipsing binaries. In contrast, Laughlin and Chambers (2002) and Nauenberg (2002) considered the stability and Doppler detectability of two planets of equal masses in a Trojan configuration, as well as other 1:1 mean-motion resonances.

Goździewski and Konacki (2006) suggested that some Trojan exoplanets (in particular, HD 128311 b & c and HD 82943 b & c) might have been mistaken for pairs in 2:1 mean-motion resonances, while Giuppone et al. (2012) found that co-orbital exoplanets could easily be confused with either single planets or 2:1 resonant pairs.

Dobrovolskis and Borucki (1996), Miralda-Escudé (2002), Agol et al. (2005), Holman and Murray (2005), and Steffen and Agol (2005) all realized that transit timing variations (TTVs) could reveal the presence of other planets in a transiting system. Later, Ford and Gaudi (2006) (see also Ford and Holman, 2007; Narita et al., 2007) derived the TTVs for a Trojan configuration of planets, and used them to place upper limits on Trojan companions of two transiting exoplanets. More recently, Madhusudhan and Winn (2009) applied the TTV technique to place upper bounds on Trojan companions of 25 such exoplanets.

This paper re-examines the detectability of Trojan systems from the standpoint of the star's reflex motion, and extends the theory to the observable effects of tides raised by the planets on the star (ellipsoidal variations). I conclude with a discussion of possible scenarios for the formation of co-orbital exoplanets. An appendix also derives an efficient new method for combining tidal perturbations from two or more sources, analogous to the composition of vectors.

2. Trojan systems

Consider the classic circular Three-Body Problem: three objects (stars, planets, satellites, or some combination) of masses m_a, m_b, and m_c, all orbiting their mutual center of mass at the vertices of an equilateral triangle of constant side r (see Fig. 1). It has long been known that this configuration is a state of dynamical equilibrium, *independent* of the individual mass values (Lagrange, 1772).

I use G for the Newtonian constant of gravitation, and $M \equiv m_a + m_b + m_c$ for the total mass of the system. Then its orbital mean motion, or angular speed, is

$$n_0 = \sqrt{GM/r^3}. \quad (1)$$

The corresponding orbital period is $P_0 = 2\pi/n_0$. If $m_c = 0$, for example, note that Eq. (1) above reduces to Kepler's third law $n_0^2 r^3 = G[m_a + m_b]$ from the more familiar two-body problem.

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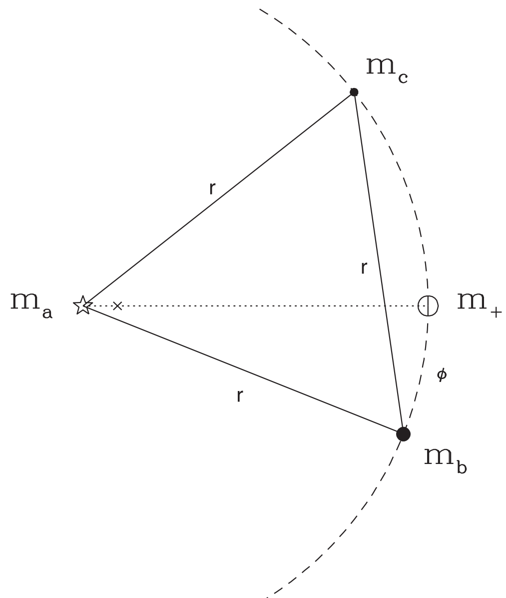

Fig. 1. Geometry of a Trojan system. In this example, a star of mass m_a and two planets of masses m_b and m_c at the vertices of an equilateral triangle of side r are all orbiting their mutual center of mass at X. The resulting reflex motion of the star is the same as if it were accompanied by only a single planet of mass $m_+ < m_b + m_c$ at a distance $\approx r$, but at an angle ϕ from m_b and $60° - \phi$ from m_c.

2.1. Stability

The criterion for stability of the equilateral configuration is more complicated:

$$m_a m_b + m_a m_c + m_b m_c < M^2/27 \tag{2}$$

(Routh, 1875). In order to interpret this inequality, it proves convenient to choose the greatest mass as m_a (without loss of generality), and to express formula (2) above in terms of two mass ratios. But rather than the ratios m_b/m_a and m_c/m_a, or even m_b/M and m_c/M, the stability criterion becomes simplest when expressed in terms of $\mu \equiv (m_b + m_c)/M$ and $f \equiv m_c/(m_b + m_c)$. This usage of μ is fairly conventional, and f was used also by Ford and Gaudi (2006), who called it ϵ. Note that by these definitions $0 \leqslant \mu \leqslant 2/3$ and $0 \leqslant f \leqslant 1$. In terms of these parameters, $m_a = (1-\mu)M$, $m_c = f\mu M$, $m_b = (1-f)\mu M$, and criterion (2) above states that the equilateral configuration becomes unstable when μ equals or exceeds the critical value

$$\mu_0 = 2\Big/\left(27 + \sqrt{27[23 + 4f - 4f^2]}\right). \tag{3}$$

The shallow solid curve in Fig. 2 (far left-hand scale) graphs μ_0 as a function of f from Eq. (3) above. Note that μ_0 is quite insensitive to f, varying by only ~1% as f ranges between 0 and 1. For $f = 0$, m_c vanishes, while m_b vanishes for $f = 1$; in either case, μ_0 attains its maximum value $\left(1 - \sqrt{23/27}\right)/2 \approx 0.03852 \approx 1/25.96$, recovering the classic result from the restricted case of the circular Three-Body Problem. However, the general case of three non-zero masses is slightly less stable, and μ_0 attains its minimum value $\left(2 - 2\sqrt{8/9}\right)/3 \approx 0.03813 \approx 1/26.23$ for $f = 1/2$ ($m_b = m_c$; see also Laughlin and Chambers, 2002).

For small displacements from equilibrium, the angle between m_b and m_c librates (oscillates) about 60°, with a combination of two periods: one slightly longer than the orbital period P_0, and another much longer. In the restricted case ($f = 0$ or 1), this longer period becomes $\sim 2P_0/\sqrt{27\mu}$ for small μ, while both periods approach $P_0\sqrt{2}$ as μ approaches its critical value μ_0 (Yoder et al., 1983; Murray and Dermott, 1999; Nauenberg, 2002).

It is interesting to interpret this critical mass ratio μ_0 in terms of the secondaries' Hill spheres, roughly equivalent to their Roche lobes. The Hill sphere sometimes also is called the "sphere of influence", or "activity sphere", although both of these terms properly refer to a different concept; see Danby (1962, p. 268). The classic Hill sphere of a secondary of mass m at a distance r from a primary of much greater mass m_a is defined as the sphere of radius

$$R_H \equiv r\left[\frac{m}{3m_a}\right]^{1/3} \tag{4}$$

centered on the secondary. It roughly describes the region in which the gravitational attraction of the secondary dominates the tidal perturbation from the primary.

Based on results from Gladman (1993), Chambers et al. (1996) have defined a "mutual" Hill radius of two secondaries of masses m_b and m_c at distances r_b and r_c from a primary of mass m_a as

$$R_\mu = \frac{r_b + r_c}{2}\left[\frac{m_b + m_c}{3m_a}\right]^{1/3}. \tag{5}$$

See Smith and Lissauer (2009, 2010) for a slight improvement to this definition; see also Salo and Yoder (1988), Hasegawa and Nakazawa (1990), Kokubo and Ida (1995, 1998), Ito and Tanikawa (1999), Marzari and Weidenschilling (2002), and Chatterjee et al. (2008). However, I expect that it would be more meaningful to weight the distances by the masses, and to define the mutual Hill radius as

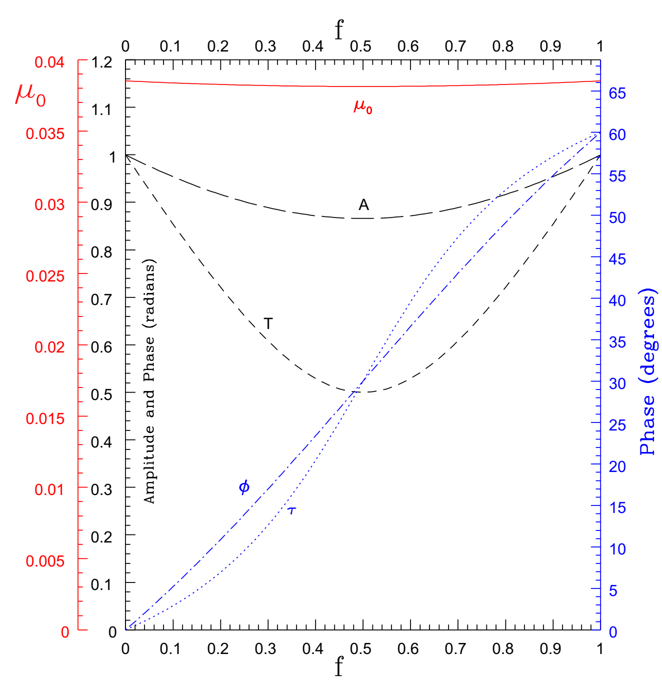

Fig. 2. Various functions of the parameter f. Solid curve (outer left-hand scale): critical mass ratio μ_0 for stability of a Trojan system. Long-dashed hyperbola (inner left-hand scale): normalized amplitude A of radial velocity variation. Short-dashed hyperbola (inner left-hand scale): normalized amplitude T of tidal perturbation. Dot-dashed curve: phase ϕ of Doppler signal, in degrees (right-hand scale) or radians (inner left-hand scale). Dotted curve: phase τ of tidal perturbation, in degrees (right-hand scale) or radians (inner left-hand scale). Note that A,T, and μ_0 are symmetric about the vertical mid-line $f = 1/2$, while ϕ and τ are point-symmetric about $f = 1/2$, $\phi = \tau = 30° = \pi/6 \approx 0.5236$ radians; thus $\phi(1-f) = 60° - \phi(f)$ and $\tau(1-f) = 60° - \tau(f)$.