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Stability of co-orbital ring material with applications to the Janus–Epimetheus system

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ABSTRACT

An analytical model that describes the evolution of ring particles that are co-orbital with two larger bodies on near-circular and near-planar orbits has been formulated. This can be used to estimate the lifetime of the particles within the ring. All the examples investigated, such as the Janus–Epimetheus (JE) system, indicate that the particles should be removed from the co-orbital region within half a synodic period (~4 years for JE). Numerical modelling confirms the predictions of the model. When the masses are on eccentric orbits the particles remain within the co-orbital system for longer. Our results suggest that the ring associated with Janus and Epimetheus must be continually fed with material, probably by meteoroid impacts on the two satellites.

1. Introduction

Analysis of the circular restricted three-body problem has produced insights that explain and predict many of the dynamical features of the Solar System. The problem consists of a primary and secondary mass moving around their common centre of mass on circular orbits while a third object, a test particle, moves within the system governed only by the gravitational attraction of the two masses. When a rotating reference frame is applied to the system, which has its origin at the centre of mass and rotates at the same angular velocity as the two masses, five equilibrium points, known as the Lagrangian points are obtained. When the mass of the primary object is much larger than the mass of the secondary object, all of these points lie close to the radius at which the secondary object orbits the primary (Danby, 1988; Roy, 1988; Murray and Dermott, 1999).

The linear stability of the L_4 and L_5 triangular Lagrangian points at low secondary: primary mass ratios ($\lesssim 0.0385$ which means it is valid for nearly all Solar System applications) allows the particle to perform 'tadpole' orbits around each of these points (Murray and Dermott, 1999). This motion is a combination of a long period motion of an epicentre librating around the triangular equilibrium point and a short period motion of the particle moving around the epicentre. The timescale of the latter motion is close to the orbital period of the secondary object. When the eccentricity and inclination of the particle's orbit around the primary mass is zero, the amplitude of the epicyclic motion is zero and the particle remains at the epicentre (Murray and Dermott, 1999).

Increasing the radial separation between the epicentre's path and the triangular equilibrium point causes the path to become more elongated around the radius of the secondary until it forms the 'critical tadpole' path at which its 'head' and 'tail' have azimuthal separations of 23.9° and 180° from the secondary respectively (Murray and Dermott, 1999). Increasing the radial separation further produces 'horseshoe' shaped orbits in which the path fully encompasses the L_3, L_4 and L_5 points. These had been analytically predicted by Brown (1911) but had not been detected within the Solar System until the discovery of Janus and Epimetheus (Dollfus, 1966; Walker, 1966; Fountain and Larson, 1978; Pascu, 1980; Pascu et al., 1980). Increasing the radial separation further produces open epicyclic paths such as 'non-symmetrical horseshoe' encounters and then chaotic motion as the particle has close encounters with the secondary.

There are many examples within the Solar System of co-orbital motion. Numerical integrations have indicated that some asteroids have co-orbital orbits with the planets of the inner Solar System. For example, the Asteroids 2002 VE68, 3753 Cruithne and 5261 Eureka perform co-orbital motion with Venus, Earth and Mars respectively (Mikkola et al., 1994a,b; Michel, 1997; Wiegert et al., 1997, 1998). An example of tadpole co-orbital motion are the Trojan asteroids located near Jupiter's L_4 and L_5 points. Further examples of tadpole co-orbital motion are within the saturnian system where Helene and Polydeuces librate around the L_4 and L_5 points of Dione respectively (Lecacheux et al., 1980; Reitsema

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et al., 1980; Murray et al., 2005) while Telesto and Calypso are in tadpole orbits co-orbital with Tethys (Seidelmann et al., 1981). The dynamically-rich saturnian system is further enhanced by the satellites Janus and Epimetheus which perform a variation of horseshoe motion (Harrington and Seidelmann, 1981; Dermott and Murray, 1981b). Within the radial locations of Janus and Epimetheus a diffuse ring has been recently discovered (Porco et al., 2006).

In ring systems, satellites can maintain co-orbital ring material. For example, Pan maintains a ring within the Encke Gap of the saturnian ring system (Porco et al., 2005). It is likely that the largest bodies within a typical ring can similarly maintain co-orbital ring material. If there are a number of bodies of similar mass located at a similar semi-major axis within a ring, interesting dynamical structures may be produced. All of these objects may be able to 'horseshoe' off each other; we refer to this concept as multiple horseshoe dynamics and it gives a possible explanation for time variable rings found within the Solar System. For example, Saturn's F-ring can be modelled well by a precessing ellipse (Bosh et al., 2002) and there is evidence of a number of large bodies contained within it (Esposito et al., 2008; Murray et al., 2008; Beurle et al., 2010).

Here we explore the stability of rings within a simple multiple horseshoe system. First, a general horseshoe model is described and investigated in Section 2. This will be used as a basis to generate an analytic model that calculates the trajectories of less massive additional objects that are contained within the horseshoe region of the system in Sections 3 and 4. This model can be ultimately used to calculate the dynamical lifetime of these particles (Section 5). Examples of the evolution of possible horseshoe systems are given in Section 6. The Janus–Epimetheus system is investigated in Section 7 which gives insight on the age of the ring that is associated with these satellites. Section 8 considers the case when an additional object has mass.

2. The basic horseshoe orbit model

Unlike the restricted three-body problem, a more general horseshoe system consists of two bodies which may have similar mass, m_1 and m_2 respectively ($m_1 \geqslant m_2$), orbiting a primary body of mass M. In a reference frame that is rotating with the average angular speed of the satellites, the epicentre of each satellite will move along its own horseshoe path similar to that seen in Fig. 1. Note that the figure is not to scale as the radial widths of the horseshoe orbits have been greatly enlarged. As the object on the inner semi-major axis catches up with the outer object, gravitational attraction causes the objects to 'swap' their approximate radial positions around a barycentric semi-major axis (Namouni, 1999),

$$a = \frac{a_1 m_1 + a_2 m_2}{m_1 + m_2} \tag{1}$$

where a_1 and a_2 are the semi-major axes of the two bodies m_1 and m_2 respectively. This encounter places the original inner object on a semi-major axis that is now larger than its co-orbital companion causing it to recede from it (see Fig. 1).

Here we consider the eccentricities and inclinations of all the bodies within the system to be zero such that the orbits follow the paths of their individual epicentres. The rings and satellites of the Solar System that are of interest generally have low eccentricity ($\sim 10^{-3}$) and orbit within the same approximate plane so this is an acceptable approximation for this initial analysis. As the eccentricity is an adiabatic invariant of symmetrical horseshoe motion (Hénon and Petit, 1986) all objects that undergo a symmetric horseshoe encounter will have zero eccentricity after the encounter. As we shall see, analysis shows that such a horseshoe system can be defined by just three parameters. It should also be

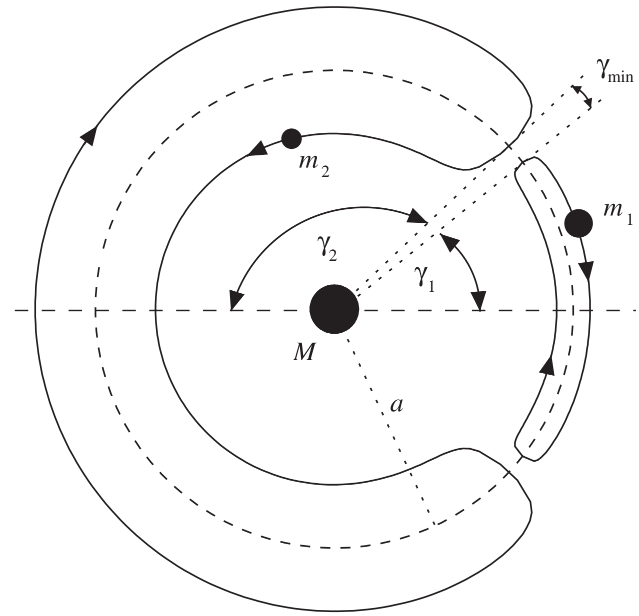

Fig. 1. A horseshoe system consists of two objects of mass m_1 and m_2 which orbit around a primary body of mass M. The motion of the two bodies is shown in a rotating reference frame in which the radial widths of the horseshoe orbits have been artificially enlarged by a factor of 1000. The frame rotates at the average angular speed of the satellites which is equivalent to the mean motion of an object placed at the semi-major axis a (shown by the dashed circle). The minimum angular separation between the satellites is $\gamma_{\min}$ while γ_1 and γ_2 define the azimuthal extent of the orbits within the rotating frame.

noted that there is a close relationship between the epicentre path and its associated zero-velocity curve (Dermott and Murray, 1981a).

Following a model similar to that used by Lissauer et al. (1985), the two bodies have longitudes θ_1 and θ_2 with respect to a reference direction (see Fig. 2). The azimuthal extent of the orbits within the rotating frame are defined by γ_1 and γ_2 while $\gamma_{\min}$ is the longitudinal minimum between the objects (see Fig. 1). When

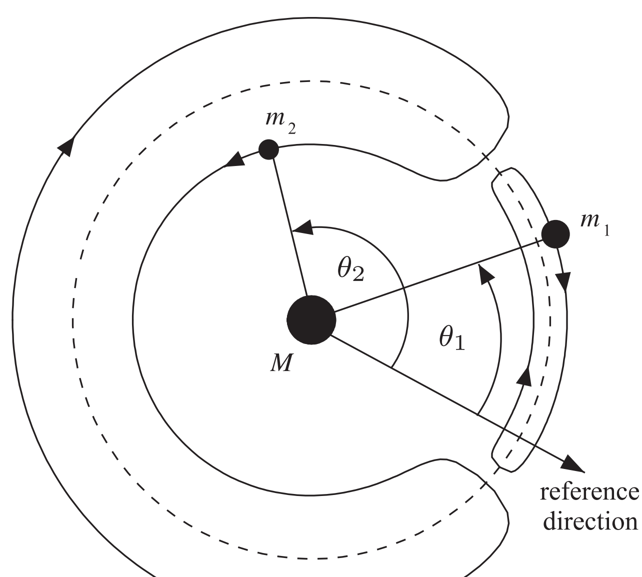

Fig. 2. The longitudes θ_1 and θ_2 (referred to a fixed direction) of the two mutual horseshoeing bodies shown in Fig. 1. The relative longitude between the bodies is defined to be $\theta_{1,2} = \theta_1 - \theta_2$.