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Adhesion and collisional release of particles in dense planetary rings

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ABSTRACT

We propose a simple theoretical model for aggregative and fragmentative collisions in Saturn's dense rings. In this model the ring matter consists of a bimodal size distribution: large (meter sized) boulders and a population of smaller particles (tens of centimeters down to dust). The small particles can adhesively stick to the boulders and can be released as debris in binary collisions of their carriers. To quantify the adhesion force we use the JKR theory (Johnson, K., Kendall, K., Roberts, A. [1971]. Proc. R. Soc. Lond. A 324, 301–313). The rates of release and adsorption of particles are calculated, depending on material parameters, sizes, and plausible velocity dispersions of carriers and debris particles. In steady state we obtain an expression for the amount of free debris relative to the fraction still attached to the carriers. In terms of this conceptually simple model a paucity of subcentimeter particles in Saturn's rings (French, R.G., Nicholson, P.D. [2000]. Icarus 145, 502–523; Marouf, E. et al. [2008]. Abstracts for "Saturn after Cassini–Huygens" Symposium, Imperial College London, UK, July 28 to August 1, p. 113) can be understood as a consequence of the increasing strength of adhesion (relative to inertial forces) for decreasing particle size. In this case particles smaller than a certain critical radius remain tightly attached to the surfaces of larger boulders, even when the boulders collide at their typical speed. Furthermore, we find that already a mildly increased velocity dispersion of the carrier-particles may significantly enhance the fraction of free debris particles, in this way increasing the optical depth of the system.

1. Introduction

Saturn's dense ring system consists of a large number of water ice particles, which, owing to frequent mutual dissipative collisions, flattened into a thin disc (Colwell et al., 2009; Cuzzi et al., 2009; Schmidt et al., 2009). Light scattering properties of the rings are consistent with a power-law size distribution $n(r) \sim r^{-\beta}$ with a slope near $\beta = 3$, ranging from centimeters to tens of meters, varying with the radial distance from the planet (French and Nicholson, 2000; Showalter and Nicholson, 1990; Zebker et al., 1985 see also Cuzzi et al., 2009; Schmidt et al., 2009). In particular, a lack of particles smaller than, roughly, 1 cm (French and Nicholson, 2000) is implied by comparison of the optical depth of the main rings at different wavelength (from microwaves to UV), and detailed photometry of the A ring (Dones et al., 1993). Such a size distribution may arise from a balance of aggregation between particles and fragmentative collisions of aggregates (Davis et al., 1984; Weidenschilling et al., 1984). The adhesive forces, leading to aggregation, are strongest for small particles (Chokshi et al., 1993). Although small debris should be steadily created in hypervelocity impacts of micrometeoroids on the rings (Cuzzi and Durisen, 1990; Durisen, 1984), it seems plausible that it is rapidly and permanently adsorbed on the surfaces of the larger particles, which would explain the small contribution of this particle size to the optical depth of the rings. This scenario was qualitatively discussed by Dones et al. (1993), who estimated the critical radius for sticking using the adhesion model proposed by Chokshi et al. (1993).

In this paper we investigate this idea quantitatively in terms of a kinetic model with a bimodal distribution of particle sizes, for binary adhesive and dissipative particle collisions. We consider consistently the kinetics of coagulation and fragmentation of particles. The balance of fragmentation of particles and their gravitational aggregation, leading to the establishment of the power-law size distribution in Saturn's rings was studied by Longaretti (1989). A simple model for ballistic aggregation and fragmentation was introduced recently by Brilliantov et al. (2009), however, without specifying a detailed mechanism of fragmentation.

We discuss the possibility that an increased velocity dispersion of the carrier-particles may significantly enhance the fraction of free debris particles, in this way increasing the optical depth of the system. Such a process may be important to understand quantitatively the observed brightness variations in perturbed ring regions, like satellite resonances (Dones et al., 1993; Nicholson et al., 2008) and the lobes of the propellers (Spahn and Sremčević,

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2000) observed in the A ring (Sremčević et al., 2007; Tiscareno et al., 2006).

The plan of the paper is as follows. In Section 2 we present the theoretical model, and calculate release and sticking rates of debris particles on the surfaces of carriers. The balance of these processes is analyzed in Section 3, establishing a steady distribution of free debris, depending on the parameters of the model. We discuss our results and their applications to Saturn's rings in Section 4, and give our conclusions in Section 5.

2. Model

We consider a bimodal system consisting of identical large boulders (carriers) of radius R and smaller particles with a range of radii $r \ll R$ (Fig. 1). Due to adhesion small particles can attach to the surface of bigger ones. We assume that boulders and debris particles are indestructible, while the fragmentation of the aggregates formed by carriers and attached debris is possible. Thus the total number of small particles of size r per unit volume $N_d(r)$ is comprised of free debris with number density $n_v(r)$ and a population covering the surface of boulders with surface density $n_s(r)$. Then the mass conservation implies:

$$N_d(r) = n_v(r) + 4\pi R^2 n_s(r) N_c, \qquad (1)$$

where N_c is the number density of carriers. We neglect the change of carrier size due to the attached debris particles.

The equilibrium number density of small particles $n_v(r)$ (debris) arises from the balance of their aggregation with boulders and detachment in collisions of carriers. The kinetic equation, describing these processes reads

$$\frac{dn_v(r)}{dt} = I_+(r) - I_-(r), \qquad (2)$$

where I_+ is debris production rate (the number density of debris of radius r, released per unit time in unit volume), I_- – their adsorption rate (the number density of smaller particles, adsorbed at the surface of carriers per unit time in unit volume). I_+ and I_- are calculated below. We neglect the interaction of debris particles with each other.

Let $\vec{V}_c$ and $\vec{V}_d$ be the velocities of carriers and debris particles measured in a frame that co-rotates with the local Keplerian speed at given distance from the central planet. We define the relative velocity of debris and carriers as $\vec{V}_{cd} = \vec{V}_c - \vec{V}_d$. Let us also denote the velocity dispersions of carriers and debris as $v_{c,d} = \sqrt{\left\langle V_{c,d}^2 \right\rangle}$.

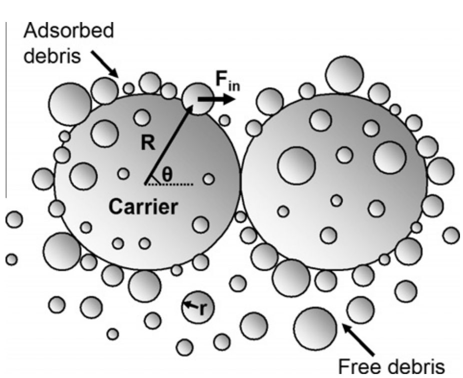

Fig. 1. A bimodal system consisting of carriers of radius $R \sim 1$ m and debris of radii $r < 10$ cm. The debris can either move freely or be attached to the surface of carriers due to adhesive forces. Debris can be released during the collision of carriers, if the normal component of inertial force, acting on debris $F_{in}\cos\theta$ (Eq. (14)) exceeds the adhesion force F_{sep} (Eq. (7)). (Here θ is the angle between the line, connecting the centres of colliding carriers and the line, connecting the centres of the carrier and the debris particle, resting on its surface.) Debris particles can be re-adsorbed in subsequent collisions.

Although the velocity distribution function of particles in Saturn's rings is anisotropic (Goldreich and Tremaine, 1978), we do not expect a significant influence of this anisotropy on the average outcome of aggregation and fragmentation. For sake of simplicity we assume here an isotropic Maxwellian distribution of random (thermal) speeds of particles

$$f\left(\vec{V}_{c,d}\right) = \left(\frac{3}{2\pi v_{c,d}^2}\right)^{3/2} \exp\left(-\frac{3}{2}\frac{V_{c,d}^2}{v_{c,d}^2}\right). \qquad (3)$$

For simplicity we assume, that the velocity dispersion v_d is equal for debris of all radii (Salo, 1992b). We also do not take into account the deviation of velocity distribution function from the Maxwellian (which corresponds to the Rayleigh distribution in orbit elements), found by Ohtsuki and Emori (2000) in the computer simulation of dense self-gravitating systems.

We assume, that all particles are composed of ice with Young modulus $Y = 7 \times 10^9$ Pa and Poisson ratio $\nu = 0.25$ (Chokshi et al., 1993). It is also common to introduce the elastic constant $D = \frac{3}{2}\frac{(1-\nu^2)}{Y} = 2 \times -10\ \mathrm{Pa}^{-1}$. The material densities of carriers and smaller particles are also assumed to be equal: $\rho_c = \rho_d = 900$ kg/m^3.

The adhesive interactions are characterized by the adhesion coefficient γ, which is twice the surface tension coefficient. The adhesion coefficient was estimated theoretically for pure ice surfaces as $\gamma = 0.74$ N/m (Chokshi et al., 1993). Hatzes et al. (1991) have shown experimentally, that adhesive interactions of ice particles significantly depend on their surface structure. In the recent experiments of Gundlach et al. (2011) the adhesion coefficient of micrometer-sized ice particles was found to be $\gamma = 0.19$ N/m.

Adhesive interactions become important only in the final phase of a collision, while generally the impact phase is governed by elastic and dissipative forces. The latter tend to damp the relative motion. These dissipative losses are usually quantified by the restitution coefficient, defined as the ratio of the normal components of the relative velocity after and before an impact: $\varepsilon = \left|\left(\vec{V}'_{cd}\cdot\vec{e}\right)/\left(\vec{V}_{cd}\cdot\vec{e}\right)\right|$, where $\vec{V}'_{cd}$ is the relative velocity after the collision and $\vec{e}$ is a unit vector, along the line, joining centres of particles during their impact. In general, the restitution coefficient is velocity-dependent (Bridges et al., 1984; Brilliantov et al., 1996; Ramirez et al., 1999), but in the present study we assume it to be constant. We take here $\varepsilon \simeq 0.3$ as a typical value for the restitution coefficient for ice particles, colliding with relative velocities $\sim$1 cm/s and effective radii $\sim$1 cm (Bridges et al., 1984).

2.1. Debris adsorption rate

If $\vec{V}_{cd}$ is small enough, carriers and debris can form aggregates due to adhesion. The adhesive interaction can be described in the framework of JKR theory (Johnson et al., 1971).

The force, acting during the collision of a boulder with a smaller particle, consists of two parts (Johnson et al., 1971):

$$F_a = \frac{a^3}{DR_{cd}} - \sqrt{\frac{6\pi\gamma}{D}}a^{3/2}. \qquad (4)$$

The first term corresponds to the Herzian elastic force, the second one – to adhesion. Here a is the contact radius of the colliding particles, $R_{cd} = Rr/(R+r)$ – their effective radius.

The work, required to release a bound debris particle from a carrier surface, can be written in the form (Brilliantov et al., 2007; Brilliantov and Spahn, 2006):