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Internal forcing of Mercury's long period free librations

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ABSTRACT

Observations of Mercury's spin rate do not rule out that, in addition to the forced 88-day mantle libration induced by solar torques, a decadal timescale libration may also be present. It has been proposed that this signal represents an amplified forced libration caused by the periodic 11.86 yr perturbation on Mercury's orbit by Jupiter. Here, we investigate the possibility that a decadal libration may be produced by a forcing internal to the planet. Our mechanism is based on the presence of time-dependent zonal flows generated by turbulent convection in Mercury's fluid core. Through electromagnetic coupling, these flows entrain longitudinal displacements of the inner core, which then entrain mantle librations by gravitational coupling. We construct a simple model to capture this exchange of angular momentum. Although core zonal flows are expected to have a broad frequency spectrum, we show that the resulting mantle libration is dominated by the amplification that occurs at the period of the free decadal libration of the combined mantle and inner core. Our results suggest that for plausible values of Mercury's internal magnetic field, if the inner core of Mercury is large ($\geqslant$1000 km), decadal mantle libration amplitudes of the order of 1 arcsec can be generated by zonal flows of the order of 1 km yr^{-1}. Conversely, if future observations can robustly determine an upper bound on the amplitude of Mercury's decadal librations, our results can be used as constraints on the convective dynamics in Mercury's core.

1. Introduction

Mercury's rotation is in a tidally locked state in which it rotates 1.5 times around itself for each orbit around the Sun. This 3:2 spin–orbit resonance leads to periodic solar gravitational torques acting on Mercury's equatorial elliptical figure, and to a libration in longitude dominated by an 88-day periodicity, the orbital period. Librations are observed as slight deviations in the instantaneous spin rate from the mean rotation rate. Using Earth-based radar data, Margot et al. (2007, 2012) have shown that the amplitude of the 88-day forced libration is relatively large, 38.5 ± 1.6 arcsec, suggesting that only the mantle participates in the libration rather than the whole planet. This indicates a decoupling between the core and mantle which requires the outermost part of the core to be fluid. For the iron-rich core in such a small planet to have remained partially molten to this day, the presence of a light-alloying element such as sulfur is required in order to depress the melting point of the metallic mixture (Schubert et al., 1988).

In addition to the 88-day forced libration, free librations may also influence the rotational dynamics of Mercury. If Mercury's core is entirely fluid and decoupled from the mantle, only one such mode exists: the free longitudinal libration of the mantle about the Mercury–Sun line at the perihelion of Mercury's orbit (Peale, 2005). In order to match the observed amplitude of the 88-day libration, the period of this free mantle libration is predicted to be 11.66 yr (Margot et al., 2012).

If a solid inner core is present, as is expected from planetary cooling (Solomon, 1976), a longitudinal misalignment between the elliptical figures of the mantle and inner core results in a gravitational restoring torque between the two. This leads to a modification of the free mantle libration which now also involves the inner core. When the inner core is larger than approximately 1000 km in radius, its moment of inertia is no longer negligible with respect to the mantle, and this can lengthen the free mantle-inner core (MIC) libration period to 19 yr (Dumberry, 2011; Van Hoolst et al., 2012). A second free libration exists, the out-of-phase free gravitational oscillation between the mantle and inner core. The period of this mantle-inner core gravitational (MICG) mode sits close to the period of free gravitational oscillation in the absence of solar torques and can vary between 3 and 8 yr depending on model parameters (Dumberry, 2011; Van Hoolst et al., 2012).

Since the presence of a large inner core disturbs the decadal timescale libration dynamics, this opens the possibility of constraining the size of the inner core on the basis of long-period libration observations. The most recent analysis of libration observations suggests that the amplitude of a long period libration is likely smaller than 2 arcsec, as its addition does not significantly improve the fit to observations (Margot et al., 2012). However, the

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currently available data only cover a time window of 10 yr; robust conclusions on the amplitude (and period) of a long period libration must await future observations.

In the absence of an ongoing excitation mechanism, any free librations are expected to be completely attenuated (Peale, 2005). However, if a forcing acts on a timescale close to the period of a free mode, resonant amplification can occur. One possible forcing mechanism is due to the periodic perturbations on Mercury's orbit by other planets. Notably, Jupiter, with an orbital period of 11.86 yr, can lead to a forced libration amplitude in excess of 40 arcsec due to its proximity to the free mantle libration (Peale et al., 2009; Yseboodt et al., 2010). However, such calculations do not include an inner core: since a large inner core displaces the free libration period further away from Jupiter's orbital period, the amplitude of this 11.86 yr libration may be much smaller.

In this work, we investigate whether a long-period libration may instead result from an internal forcing. The basic scenario we propose is based on the presence of time-dependent, axially-symmetric, longitudinal flows (zonal flows) in Mercury's fluid core. Zonal flows are naturally produced by turbulent rotating convection in spherical shell geometries, for both thermal (Christensen, 2002) and magnetohydrodynamic convection (Aubert, 2005). The fluctuating nature of turbulent convection results in temporal variations in the force balance underlying the zonal flows and thus, in their time-dependency (Aubert, 2005). The most likely explanation for Mercury's global, mainly dipolar magnetic field (Anderson et al., 2011) is that it is generated and maintained by convective motion in its fluid core, so time-dependent zonal flows are expected to be present. Indeed, numerical dynamo experiments directly aimed at explaining Mercury's magnetic field all contain zonal flows (Stanley et al., 2005; Christensen, 2006; Vilim et al., 2010).

Although their precise characteristics depend on factors such as inner core size, buoyancy forcing and the amplitude of the internal magnetic field, time-dependent zonal flows in Mercury's fluid core will affect its angular momentum dynamics. Through electromagnetic (EM) coupling, these flows should entrain longitudinal displacements of the inner core (Gubbins, 1981). The latter will then entrain mantle librations by gravitational coupling (Buffett, 1996). We expect that when the timescale of zonal flow fluctuations is close to the period of a free mode of libration, amplification by resonance will occur. Our goal is to establish the conditions for such a mechanism to generate a mantle libration of observable amplitude. Conversely, if future observations can place a robust upper bound on the amplitude of decadal librations, our results can be used as constraints on the amplitude of zonal flows in Mercury's core.

2. Theory

2.1. Long-period libration dynamics

The libration of Mercury's mantle γ_m is defined as the longitudinal angle which separates its axis of minimum moment of inertia from its angle had Mercury been rotating uniformly about its spin axis. The libration of the inner core γ_s is defined similarly. Peale et al. (2002) presented the equations that govern the combined libration of Mercury's mantle and solid inner core. Here, we focus on the long-period librations, for which the short-period ($\leqslant$88 day) fluctuations are averaged out. In other words, we only consider the mean solar gravitational torque over one orbit (Peale et al., 2009). In this situation, and when only gravitational forces are included, the coupled differential equations that govern the libration of the mantle and inner core are (Dumberry, 2011)

$$\frac{d^2\gamma_m}{dt^2} = -3n^2 G_{201}(\Delta I_m + \Delta I_f)\gamma_m - \frac{2\overline{\Gamma}}{C_m}(\gamma_m - \gamma_s), \tag{1}$$

$$\frac{d^2\gamma_s}{dt^2} = -3n^2 G_{201}\alpha\Delta I_s\gamma_s + \frac{2\overline{\Gamma}}{C_s}(\gamma_m - \gamma_s). \tag{2}$$

The first terms on the right-hand sides of each of Eqs. (1) and (2) capture the effects of the mean solar gravitational torque, while the second terms embody the gravitational torque between the asymmetric mantle and solid inner core. Further details on the derivations of the above equations are given in Peale et al. (2002, 2009), Dumberry (2011) and Van Hoolst et al. (2012).

ΔI_m, ΔI_s and ΔI_f in Eqs. (1) and (2) represent ratios of the principal moments of inertia ($A < B < C$) of the mantle (subscript m), inner core (subscript s), and of a body with the same shape as the core-mantle boundary (CMB) and uniform density equal to that of the fluid just under the CMB (subscript f). They are defined as

$$\Delta I_m = \frac{B_m - A_m}{C_m}, \quad \Delta I_f = \frac{B_f - A_f}{C_m}, \quad \Delta I_s = \frac{B_s - A_s}{C_s}. \tag{3}$$

Here, we follow the approach of Veasey and Dumberry (2011); Dumberry (2011) and Van Hoolst et al. (2012) and consider a three-layer model of Mercury, where the inner core (radius r_s), outer core (outer radius r_f), and mantle (outer radius r_m) each have uniform densities of ρ_s, ρ_f and ρ_m, respectively. All asymmetrical densities are modelled in terms of the equatorial ellipticities of the external surface ϵ_m, the CMB ϵ_f, and the inner-core boundary (ICB) ϵ_s. In such a model,

$$\Delta I_m = \frac{\epsilon_m - \epsilon_f\eta_f}{1 - \eta_f}, \tag{4}$$

$$\Delta I_f = \epsilon_f\eta_f\frac{\rho_f}{\rho_m}\frac{1}{1 - \eta_f}, \tag{5}$$

$$\Delta I_s = \epsilon_s, \tag{6}$$

where

$$\eta_f = \left(\frac{r_f}{r_m}\right)^5. \tag{7}$$

We note that the term ΔI_f had been omitted in the studies of Veasey and Dumberry (2011) and Dumberry (2011). Its presence is necessary to account for the pressure torque by the fluid on the underside of the CMB (Van Hoolst et al., 2012).

The other parameters that enter Eqs. (1) and (2), n, G_{201}, and α are defined as:

$$n^2 = \frac{GM_\odot}{a^3}, \quad G_{201} = \left(\frac{7}{2}e - \frac{123}{16}e^3\right), \quad \alpha = \left(1 - \frac{\rho_f}{\rho_s}\right), \tag{8}$$

where n is the mean orbital frequency, a is the semi-major axis of the orbit, e is the orbital ellipticity, G is the gravitational constant, and $M_\odot$ the solar mass. The constant of gravitational coupling between the mantle and inner core, $\overline{\Gamma}$, is expressed by

$$\overline{\Gamma} = \frac{4\pi G}{5}\alpha C_s\epsilon_s[(\rho_f - \rho_m)\epsilon_f + \rho_m\epsilon_m]. \tag{9}$$

Interior models of Mercury are constructed by assuming a mantle density (ρ_m) and sulfur concentration (χ_S^{in}) of the initially entirely fluid core. Once an inner core radius is specified, we determine the equatorial ellipticities such that they are consistent with observations of $C_{22} = (0.81 \pm 0.01) \times 10^{-5}$ (unnormalized value of Smith et al. (2012)). ϵ_s is determined in terms of ϵ_m and ϵ_f by assuming that the ICB is at hydrostatic equilibrium (Veasey and Dumberry, 2011). We follow Van Hoolst et al. (2012) and further assume that $\epsilon_m = \epsilon_f$, which allows us to find a unique value of ϵ_m