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Spatial patterns of tidal heating

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ABSTRACT

In a body periodically strained by tides, heating produced by viscous friction is far from homogeneous. The spatial distribution of tidal heating depends in a complicated way on the tidal potential and on the internal structure of the body. I show here that the distribution of the dissipated power within a spherically stratified body is a linear combination of three angular functions. These angular functions depend only on the tidal potential whereas the radial weights are specified by the internal structure of the body. The 3D problem of predicting spatial patterns of dissipation at all radii is thus reduced to the 1D problem of computing weight functions. I compute spatial patterns in various toy models without assuming a specific rheology: a viscoelastic thin shell stratified in conductive and convective layers, an incompressible homogeneous body and a two-layer model of uniform density with a liquid or rigid core. For a body in synchronous rotation undergoing eccentricity tides, dissipation in a mantle surrounding a liquid core is highest at the poles. Within a soft layer (or asthenosphere) in contact with a more rigid layer, the same tides generate maximum heating in the equatorial region with a significant degree-four structure if the soft layer is thin. The asthenosphere can be a layer of partial melting in the upper mantle or, very differently, an icy layer in contact with a silicate mantle or solid core. Tidal heating patterns are thus of three main types: mantle dissipation (with the icy shell above an ocean as a particular case), dissipation in a thin soft layer and dissipation in a thick soft layer. Finally, I show that the toy models predict well patterns of dissipation in Europa, Titan and Io. The formalism described in this paper applies to dissipation within solid layers of planets and satellites for which internal spherical symmetry and viscoelastic linear rheology are good approximations.

1. Introduction

Fifty years ago, William Kaula found that the heat dissipated by tidal friction within the Moon is extremely nonuniform both radially and laterally (Kaula, 1963, 1964). At that time, nonuniform tidal heating was a rather academic subject as these variations were not observable: tidal heating in the Moon is indeed much smaller than radiogenic heating. Spatial variations of tidal heating were actually a byproduct of computing the total power dissipated in the body, a key factor in modeling orbital evolution. Kaula's calculations were based on the microscopic (or micro) approach to tidal dissipation, which starts with the computation of viscoelastic tidal strains. A bit earlier, Munk and MacDonald (1960) had given an approximate formula for the total power dissipated by tides with a macroscopic (or macro) approach. This alternative approach does not require the computation of tidal strains: the tidal bulge lags the tidal forcing by an angle parameterizing the viscous response. Their formula, however, was limited to an incompressible homogeneous body, in contrast with the micro approach which is applicable to any model of internal structure.

New theoretical developments had to wait until 1978, when two papers significantly improved tidal heating computations. First, Peale and Cassen (1978) reconsidered both macro and micro approaches to tidal heating, correcting several errors in the various formulas. They also mapped tidal heating variations for eccentricity tides and obliquity tides, showing that tidal dissipation in a homogeneous Moon is maximum at the poles in the former case and at the equator in the latter. Furthermore they showed that the presence of a large liquid core enhances dissipation in the mantle. Second, Zschau (1978) derived a simple formula for the total dissipated power in a spherically stratified compressible body, in which the influence of the body's internal structure appears through $Im(k_2)$, the imaginary part of the gravity tidal Love number k_2. Zschau's formula was the first step toward reconciling the macro and micro approaches, since k_2 can be computed if the internal structure of the body is known. Tobie et al. (2005b) later applied the variational method to relate $Im(k_2)$ to the imaginary part of the volume-integrated strain power. They also computed the radial distribution of the dissipated power in terms of deformation functions that can be evaluated with standard methods for any

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spherically stratified model. I will show that the formulas of Zschau (1978) and Tobie et al. (2005b) are recovered by spatially averaging the local power obtained in the micro approach, thus bridging the last gap between the macro and micro approaches. My paper, however, is primarily about the angular distribution of the dissipated power.

Spatial variations in tidal heating became relevant when spacecraft sent back incredible surface data showing that the Galilean satellites Io and Europa undergo strong tidal deformations and heating. Tidal heating was the only explanation for Io's volcanism (Peale and Cassen, 1978) but what was going on beneath the surface was a mystery. Maybe the distribution of volcanoes could be used in order to constrain the internal structure of the satellite? Segatz et al. (1988) suggested that dissipation occurred either in the whole mantle or mostly in a thin asthenosphere close to the surface. These two models predict completely different patterns of surface heat flux with maximum dissipation at the poles in the former case and at the equator in the latter (a mix of the two is of course possible). Galileo data have often been interpreted as favoring the asthenospheric dissipation model though the issue remains controversial (Lopes-Gautier et al., 1999; Tackley, 2001; Kirchoff et al., 2011; Veeder et al., 2012; Hamilton et al., 2012). Another idea consists in using long wavelength topography as an indirect measure of the heat flux (valid if the topography is isostatically compensated), but the promising analysis of Voyager data (Ross et al., 1990) was not confirmed by Galileo measurements (Thomas et al., 1998). Lateral variations of surface heat flux however also depend on the heat transport mechanism which could be determinant.

On icy satellites like Europa, Titan and Enceladus, spatial variations of tidal heating are interesting for other reasons. In these satellites, tidal heating can melt the ice at depth and create a global subsurface ocean. The covering icy shell varies in thickness because of spatial variations in tidal heating and solar insolation. Ojakangas and Stevenson (1989a,b) computed icy shell thickness variations on Europa with the aim of predicting nonsynchronous rotation and polar wander. Nimmo et al. (2007) and Nimmo and Bills (2010) used the same method to predict long wavelength topography on Europa and on Titan, respectively. Variations of the thickness of the icy shell (or more generally the lithosphere) also influence surface tectonics (Beuthe, 2010).

Finally, spatial variations of tidal heating are important because of their strong coupling to convection. Several convection models have used as input tidal heating predicted by spherically stratified models (Tackley, 2001; Tobie et al., 2003; Roberts and Nimmo, 2008). An important limitation of this approach is the neglect of lateral viscosity variations on tidal heat production, which can be taken into account by giving up the assumption of spherical symmetry and solving simultaneously for convection and tidal dissipation (Běhounková et al., 2010; Han and Showman, 2010). Chaos terrain on Europa could be a visible result of spatially varying tidal heating enhanced by a local drop in viscosity.

Until now, predicting spatial patterns of tidal dissipation meant computing the dissipated power at every point within the body. This laborious procedure obscures the link between the internal structure and the resulting pattern, and makes it difficult to look for all possible patterns generated by realistic internal structures. In this paper, I show that the dissipated power at a given radius within a spherically stratified body is the linear combination of three basic patterns that depend only on the tidal potential. The coefficients weighting the patterns depend are radial functions which can be computed with standard methods developed for tidal deformation problems once the internal structure of the body has been specified.

I study the influence of the internal structure on dissipation patterns by computing the dissipation weight functions in various toy models without assuming a specific rheology. The toy models are the thin icy shell above an ocean, the homogeneous body (relevant to a completely solid body with little stratification or to a solid core) and the incompressible two-layer body, either with a liquid core or with rigid core, the latter case being relevant to dissipation in a soft layer (such as an asthenosphere) above a more rigid layer. It is well-known that dissipation patterns are completely different if dissipation occurs in the deep mantle and in a thin asthenosphere. Besides these two classes of patterns, I show that dissipation in a thick asthenosphere leads to a third type of dissipation pattern, with maximum heating at the equator as in asthenospheric dissipation but with a lower content in harmonic degree four. Toy models predict well dissipation patterns within real bodies though not their magnitude. I will give three examples of this by computing dissipation weight functions for realistic internal structures of Europa, Titan and Io.

2. Dissipated power

2.1. Power, strains and tidal potential

In the micro approach to dissipation, the dissipated power within the planet or satellite is expressed in terms of tidal strains (this formula is derived in Appendix A and compared with other expressions found in the literature). If tides operate at only one angular frequency ω, the dissipated power per unit volume averaged over one orbital period is given by

$$P = \omega\, Im(\tilde{\mu})\left(\tilde{\epsilon}_{ij}\tilde{\epsilon}^{*}_{ij} - \frac{1}{3}|\tilde{\epsilon}|^2\right) + \frac{\omega}{2} Im(\widetilde{K})|\tilde{\epsilon}|^2, \tag{1}$$

where $\tilde{\mu}$ (resp. $\widetilde{K}$) is the complex shear (resp. bulk) modulus, $\tilde{\epsilon}_{ij}$ is the Fourier transform of the strain tensor and $\tilde{\epsilon}$ is the trace of $\tilde{\epsilon}_{ij}$. All quantities implicitly depend on the frequency ω and on the point $\mathbf{x}$ within the planet where the power is evaluated. In this paper, the tilde on viscoelastic parameters indicates that they are complex and frequency-dependent (the tilde is dropped if the parameters are purely elastic).

If there are several tidal frequencies, the total power averaged over time is a sum over these frequencies (interferences vanish, see Eq. (68)) so that it becomes essential to know the frequency dependence of the viscoelastic parameters (i.e. the rheology). Unfortunately the rheology of planetary bodies is poorly constrained (Jackson, 2007; Karato, 2008). Earth's mantle has been mainly studied at frequencies that are much higher (laboratory experiments), moderately higher (seismic attenuation and seismic anisotropy) or much lower (Chandler wobble and postglacial rebound) than tidal frequencies (Karato, 2010). It is indeed difficult to determine Earth's viscous response at tidal frequencies (e.g. Benjamin et al., 2006; Nakada and Karato, 2012) and even more so for other bodies. Maxwell rheology is often used in studies of tidal dissipation because it is the simplest model in which the response changes from elastic to viscous as the frequency decreases. It is however not clear how the Maxwell viscosity is related to the true viscosity of the material (Ross and Schubert, 1986; Bills et al., 2005; Sotin et al., 2009). Rheological models depending on more parameters such as the Andrade model (Castillo-Rogez et al., 2011) or the extended Burgers model (Nimmo et al., 2012) could be more realistic. Moreover there has been a long-standing debate on whether viscous deformations in Earth's mantle are mainly due to diffusion creep or to dislocation creep, corresponding to a linear or nonlinear rheology, respectively (Karato and Wu, 1993). In this paper, I assume that the rheology is linear without being more specific about it except in applications to real bodies for which I use the Maxwell model. Besides I consider only the dominant tidal frequency; this restriction is appropriate for