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The web of three-planet resonances in the outer Solar System

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Abstract

In this paper we numerically detect the web of three-planet resonances (i.e., resonances among mean anomalies, nodes and perihelia of three planets) with respect to the variation of the semi-major axis of Saturn and Jupiter, in a model including the planets from Jupiter to Neptune. The measure confirms the relevance of these resonances in the long-term evolution of the outer Solar System and provides a technique to identify some of the related coefficients.
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1. Introduction

The problem of the stability of our Solar System is one of the main problems of Celestial Mechanics. In the last decades this problem has been studied by means of long-term numerical integrations (Sussman and Wisdom, 1988, 1992; Laskar, 1989, 1996; Nobili et al., 1989) which demonstrated that while the system does not show significant instabilities (especially for the outer planets), it is nevertheless chaotic. Laskar (1990) found secular resonances responsible for the chaos of the inner planets. The chaotic nature of the outer planets is instead a problem which is still under study, with a recent advancement on its comprehension made by Murray and Holman (1999). Sussman and Wisdom (1992) remarked that long-term integrations of the outer Solar System made with different integrators and different integration steps agreed that the system is chaotic, but gave substantially different Lyapunov times. They conjectured that these discrepancies could be due to a high sensitivity of the dynamics of the system on the initial conditions, so that little changes in the integration scheme could potentially give different results. Murray and Holman (1999) found that the chaotic nature of the solutions is very sensitive to small changes of the semi-major axis of Uranus, and identified the three-body resonances among Jupiter, Saturn and Uranus as the main responsible for this. They also provided a heuristic model to show that these resonances can produce chaotic diffusion of the planets on very long times.

Three-body resonances have been already used to explain in great detail the dynamical structure of the asteroid belt (Murray et al., 1998; Nesvorny and Morbidelli, 1998a, 1998b; Morbidelli and Nesvorny, 1999). In particular, they allow one to explain much of the slow chaos arising at moderate eccentricities.

The importance of three-body resonances for the dynamics is due to the fact that though they are second-order resonances with respect to the planetary masses (they include at least three objects), and therefore their amplitude is small, they are nevertheless organized in multiplets that typically can overlap (Nesvorny and Morbidelli, 1998b) at moderate eccentricities, giving rise to chaotic motion and possibly to chaotic diffusion (for asteroids, diffusion times can be typically 100 Myr). In the planetary case, the structure of the multiplets is the following. Denoting by a_j, e_j, i_j, λ_j, ω_j, Ω_j the orbital elements of the jth planet, with j running from 5 (Jupiter) to 8 (Neptune), a three-body resonance occurs when there exist three integers n_i, n_j, n_k such that:

$$n_i\dot{\lambda}_i + n_j\dot{\lambda}_j + n_k\dot{\lambda}_k \sim 0, \tag{1}$$

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more precisely when this quantity is of the same order of the secular frequencies $\dot{\omega}_h$, $\dot{\Omega}_h$ ($h = 5, \ldots, 8$).

Therefore, as explained in Murray and Holman (1999), in the phase space of the planets there are initial conditions such that for some integer values of the k_h, k'_h the angle:

$$n_i\lambda_i + n_j\lambda_j + n_k\lambda_k + \sum_{h=5}^{8}\left(k_h\omega_h + k'_h\Omega_h\right) \tag{2}$$

is resonant, i.e., it is librating, or it is in a regime of chaotic alternation of librations and circulations. Because the secular frequencies $\dot{\omega}_h$, $\dot{\Omega}_h$ are of the same order of the planetary masses $m_5, \ldots, m_8$ (the mass of the Sun is 1; hereafter for convenience we denote also $\epsilon = m_5$), while the main contribution to the frequencies $\dot{\lambda}_i$ is given by the Keplerian approximation, for any fixed (n_i, n_j, n_k) all possible values of the k_h, k'_h generate a multiplet of resonances (2) separated in semi-major axis by order ϵ. But the strength of resonant harmonics is of order ϵ^2 (see Nesvorny and Morbidelli, 1998b; Murray and Holman, 1999, and also Section 4) so that their amplitude in semi-major axis is of order ϵ. Therefore, because the distances among these resonances and their amplitudes are of the same order, they can overlap.

To determine analytically the overlapping of these resonances is a complicated technical problem. In the case of a planar approximation of the asteroid motion an analytic approach for the precise computation of the location and separation of the different components of the multiplet has been done in Nesvorny and Morbidelli (1998b). Numerical approaches have been also used to detect the location and the structure of the resonances (Morbidelli and Nesvorny, 1999), based on the computation of the largest Lyapunov exponent of the system (actually, a Fast Lyapunov Indicator, see Froeschlé et al., 1997) with respect to the change of a critical initial condition, and then trying to associate to the peaks of positive Lyapunov exponent a linear combination of the frequencies.

In the case of the planets, the one-dimensional spans of the parameters space provided in Murray and Holman (1999) allow one to appreciate alternation of intervals of regular motions and intervals of chaotic motions, thus excluding a uniform covering of the phase space by resonance overlapping.

Moreover, two-dimensional spans of the parameters space allow one to appreciate the geometry of the resonances. In particular they allow one to appreciate if the resonances constitute a regular web (the so-called Arnold web, see Froeschlé et al., 2000) or instead there are some local overlappings. These two scenarios can have different implications on the long-term stability of the system (see Morbidelli and Guzzo, 1997; Guzzo and Morbidelli, 1997; Morbidelli, 2002).

Mitchenko and Ferraz-Mello (2001a) provide two-dimensional explorations of the phase space around each planet obtained by changing the semi-major axis and the eccentricity of one planet in each exploration. The results, obtained with 3 Myr numerical integrations, confirm that within a distance of order 10^{-2} AU from our Solar System there are some important three-planet resonances. Nevertheless, due to the limited integration time they cannot detect the three-planet resonances described by Murray and Holman (1999).

In this article, using a numerical approach based on the combined use of the fast Lyapunov indicator method (FLI hereafter) and of frequency analysis (Laskar, 1993) we will provide a direct representation of the three-planet resonances on a two-dimensional grid of initial conditions which is obtained changing the values of the initial semi-major axis of Jupiter and Saturn.

As we explain in Section 2, these are expected to be critical parameters for the exploration of the three-planet resonances in a neighborhood of the true initial condition of our Solar System.

The present exploration concerns a very small neighborhood of the true Solar System (from 2×10^{-5} to 4×10^{-3} AU), and the integration time is 20 Myr for each initial condition, so that we can detect the weak three-planet resonances which were indicated by Murray and Holman (1999) as the origin of the chaos of the outer planets.

Our results establish that in this small neighborhood of the true Solar System (obtained from JPL DE405) some families of three-planet resonances constitute an intricate web. The typical separation of these resonances is of order 10^{-5} rad/yr, and therefore a change in the semi-major axis of Jupiter (or Saturn) of about 10^{-5} AU can change the dynamical state of the system. The results agree with those of Murray and Holman (1999), who find the family of resonances $(n_5, n_6, n_7) \sim (3, -5, -7)$, but also we find other families of three-body resonances associated to the coefficients $n_6/n_5 = 3$, $11/4 \sim 2.73$, 1, 4, $17/6 \sim 2.84$, 6 (see Section 3). It is relevant that all these families are consistent with the small denominators found by Bretagnon (1981, 1982) in his analysis of the solution of the Solar System at third order in planetary masses. In Bretagnon papers the question about the location of the resonances associated to these small divisors (nor its arrangement in phase space as a web) is not raised. Nevertheless the appearance of these small denominators in the analytic solution suggests that they should be in some sense relevant to the dynamics of the Solar System, as it is precisely shown in this paper.

The position of the true Solar System with respect to the web of three-planet resonances is a peculiar one. In fact, the true Solar System is at the edge of a crossing of these resonances, and therefore its dynamical state is particularly sensitive to very small changes of the initial conditions. The computation of the Lyapunov exponent exhibits the puzzling dependence on the integration step and on the initial conditions which was first described by (Sussman and Wisdom, 1992). This can be possibly due to intermittency phenomena, which usually characterize the border of resonance crossings. This aspect will be investigated in a forthcoming paper.

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