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Cluster temperatures and non-extensive thermo-statistics

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Abstract

We propose a novel component to the understanding of the temperature structure of galaxy clusters which does not rely on any heating or cooling mechanism. The new ingredient is the use of non-extensive thermo-statistics which is based on the natural generalization of entropy for systems with long-range interactions. Such interactions include gravity and attraction or repulsion due to charges. We explain that there is growing theoretical indications for the need of this generalization for large cosmological structures. The observed pseudo temperature is generally *different* from the true thermodynamic temperature, and we clarify the connection between the two. We explain that this distinction is most important in the central part of the cluster where the density profile is most shallow. We show that the observed pseudo temperature may differ up to a factor 2/5 from the true thermodynamic temperature, either larger or smaller. In general the M–T and L–T relations will be affected, and the central DM slope derived through hydrostatic equilibrium may be either more shallow or steeper. We show how the true temperature can be extracted correctly either from the spectrum or from the shape of the Doppler broadening of spectral lines.

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1. Introduction

Galaxy clusters have been known and studied for many years, and the radial dependence of cluster temperatures is becoming a testing ground for models of structure formation and for our understanding of gas dynamics. The emerging temperature profile is one where the temperature increases from the centre to some maximum temperature, and then decreases again for larger radii.

The central decrement has been much discussed and the possibility of cooling flows has been explained in excellent reviews, see (Fabian, 1994;

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Donahue and Voit, 2004) for references. From an observational point of view this central temperature decrement is very well established (Allen et al., 2001; Peterson et al., 2001; Kaastra et al., 2004), and numerical simulations are now beginning to see it too (Motl et al., 2004). The outer temperature decrease is well established both observationally (Markevitch et al., 1998; Kaastra et al., 2004) and numerically (Loken et al., 2002; Komatsu and Seljak, 2001).

The expected cooling flows are not observed in galaxy clusters, and numerous explanations have been proposed including various additional heating or lack-of-cooling mechanisms, see e.g. (Peterson et al., 2001; Donahue and Voit, 2004) for references.

We will here propose a new element in the understanding of cluster temperatures. Our solution has galaxy clusters in *kinetic equilibrium*, but with temperatures defined through non-extensive thermo-statistics which is the natural generalization of normal statistical mechanics. The need for non-extensive entropy arises when particle interactions are not point-like, and includes gravity and attraction/repulsion due to charges. These are exactly the conditions for electrons and protons in galaxy clusters, as we will explain in Section 4.

We will structure the paper as follows. First we will consider the theoretical basis for statistical mechanics with non-extensive entropy. To clarify the signatures of non-extensive statistics, we will apply the results to the Coma cluster. This is followed by a discussion of reasons for using Tsallis statistics, potential problems and implications of our findings. Finally we offer our conclusions.

2. Tsallis statistics

Let us first consider the theoretical basis for statistical mechanics with non-extensive entropy. Statistical mechanics for classical gases can be derived from the Boltzmann–Gibbs assumption for the entropy, $S_{\mathrm{BG}} = -k\sum p_i \cdot \ln p_i$, where p_i is the probability for a given particle to be in the state i, and the sum is over all states. For normal gases the probability, $p(v)$, coincides with the velocity distribution function, $f(v)$. This classical statistics can be generalized to Tsallis (also called non-extensive) statistics (Tsallis, 1988), which depends on the entropic index q

$$S_q = -k\sum_i p_i^q \cdot \ln_q p_i. \tag{1}$$

Here the q-logarithm is defined by, $\ln_q p = (p^{1-q} - 1)/(1-q)$, and for $q = 1$ the normal Boltzmann–Gibbs entropy is recovered, $S_{\mathrm{BG}} = S_1$. The probabilities still obey, $\sum p_i = 1$, while the particle distribution function is now given by $f(v) = p^q(v)$. Thus, for $q < 1$ one privileges rare events, whereas $q > 1$ privileges common events. For a summary of applications see (Tsallis, 1999), and for up to date list of references see http://tsallis.cat.cbpf.br/biblio.htm.

Average values are calculated through the particle distribution function, and one e.g., has the mean energy (Tsallis et al., 1998)

$$U_q = \frac{\sum p_i^q E_i}{c_p}, \tag{2}$$

where $c_p = \sum p_i^q$, and E_i are the energy eigenvalues. Optimization of the entropy in Eq. (1) under the constraints leads to the probability (Silva et al., 1998; Tsallis et al., 2003)

$$p_i = \frac{\left[1 - (1-q)\beta_q\left(E_i - U_q\right)\right]^{1/(1-q)}}{Z_q}, \tag{3}$$

where Z_q normalizes the probabilities, $\beta_q = \beta/c_p$, and β is the optimization Lagrange multiplier associated with the average energy. Adding a constant energy, ϵ_0, to all the energy eigenvalues leads to $U_q \to U_q + \epsilon_0$, which leaves all the probabilities, p_i, invariant (Tsallis et al., 1998). By defining

$$\alpha = 1 + (1-q)\beta_q U_q. \tag{4}$$

Eq. (3) can be written as

$$p_i = \frac{\left(1 - (1-q)(\beta_q/\alpha)E_i\right)^{1/(1-q)}}{Z'_q}, \tag{5}$$

and we see that the probabilities have the shape of q-exponential functions

$$p_i = \frac{\exp_q\left(-\beta'_q E_i\right)}{Z'_q}, \tag{6}$$