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Is hydrodynamic escape from Titan possible?

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ABSTRACT

When examining thermal atmospheric escape, usually either Jeans escape or hydrodynamic escape is considered. Jeans escape, where particles with velocities higher than the escape velocity can escape a planetary atmosphere, is usually considered, when particles in a collision-free region are examined. Hydrodynamic escape, on the other hand, presumes that the outflowing gas can be considered as a continuous, homogeneous medium where neither light, nor heavy particles can be discriminated from each other. Recently, Strobel (2009) applied a so-called 'slow hydrodynamic escape model', which describes cases intermediate between Jeans escape and hydrodynamic escape, for the nitrogen and methane molecules in Titan's upper atmosphere. This model requires an extended quasi-collisional region above the exobase where efficient energy transfer can presumably occur. In this study, we examine the collision probability of nitrogen and methane molecules with ambient atmospheric particles within Titan's exosphere using a modified Monte Carlo code introduced by Wurz and Lammer (2003), to analyze if the 'slow hydrodynamic escape model' is applicable to Titan's exosphere or not. Our results show that the collision probability of nitrogen and methane within Titan's exosphere decreases quickly with height above the exobase. Also, the probability of a nitrogen or a methane molecule to collide with another heavy molecule is far larger than the probability of a collision with a light particle, since in the region where nitrogen and methane are mainly present, the heavy molecules dominate the light molecules by a factor of 10–100. The results of our particle simulation do not confirm the existence of an extended quasi-collisional region above the exobase, where heavy, slow molecules can gain the escape velocity through collisions with light, fast particles.

1. Introduction

Ever since the ion neutral mass spectrometer (INMS) on cassini-huygens has provided us with indications of considerably larger than expected loss rates for heavy species from Titan's atmosphere (De La Haye et al., 2007b; Yelle et al., 2008), different models have tried to explain the reason for these loss rates (Strobel, 2009; Johnson, 2009). Some of these models apply non-thermal escape processes where the energy is deposited directly in the exobase region (Lammer et al., 1998; De La Haye et al., 2007a,b; Shematovich et al., 2003; Michael and Johnson, 2005; Wahlund et al., 2005). Other models treat thermal escape processes, where the energy is deposited well below the exobase and is transferred to higher altitudes through collisions and diffusion of the atmospheric particles (Cui et al., 2008; Strobel, 2008, 2009; Yelle et al., 2008). Especially Strobel (2008, 2009) introduced a hypothesis of so-called 'slow hydrodynamic escape' based on the early pioneering works of Watson et al. (1981) and Kasting and Pollack (1983) who studied hydrodynamic escape of an atomic hydrogen rich thermosphere from a terrestrial planet due to solar EUV heating by applying idealized hydrodynamic equations. From their thermospheric model these authors obtained supersonic flow solutions for which the sonic point was reached at a distance of about 30 planetary radii. These authors argued that supersonic hydrodynamic escape of atomic hydrogen was possible from Earth's atmosphere if it was dominated by hydrogen even if it was exposed to the present time Sun's EUV flux. However, as pointed out by Lammer et al. (2008), these studies assumed that the hydrodynamic fluid equations applied above the exobase level are not internally self-consistent and atom velocities at the exobase level were much lower than the escape velocity. The question remains if the atoms can really achieve their supersonic speed and 'slow hydrodynamic escape' can occur. In the past and current year, several Monte Carlo simulations were conducted to reassess the applicability of the models treating thermal escape processes (Johnson, 2009, 2010; Tucker and Johnson, 2009; Volkov et al., 2011). The findings of these simulations lead to the conclusion that the applied thermal

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hydrodynamic escape models are inapplicable to Titan's exosphere. The major problem arises when trying to confirm the needed energy transfer from lighter, faster particles to heavier, slower particles within Titan's exosphere. The aim of this study is to evaluate the collision probabilities of N_2 and CH_4 molecules in Titan's exosphere, since this transfer presumably occurs through thermal conduction. We apply a Monte Carlo code to investigate if the conversion of internal thermal energy of the neutral gas into kinetic energy of the flow is retarded by the lack of collisions above the exobase. If the flow of neutral particles cannot be accelerated anymore, it is not clear that either the sonic or escape velocity can be reached. In Section 2 we describe the applied Monte Carlo model, in Section 3 we present our results and in Sections 4 and 5 we discuss our results.

2. Model

The exobase is usually defined as the level, where the mean free path is equal to the local scale height. Strictly speaking, the exobase is not a line but a region, and its level should be different for each species, due to their different characteristics. In this study we evaluate the distance where collisions can still occur above the exobase level. We initialize 10^6 methane and 10^6 nitrogen molecules and follow each particle separately along its individual trajectory from the exobase up to a height of 20 times the exobase scale height. The exobase height was set equal to 1429 km and the temperature was set to be 149 K (both values are taken from Waite et al., 2005). These values are in good agreement with the exobase height and exobase temperature also calculated from the INMS data by different authors. De La Haye et al. (2007b) calculate the exobase height as being between 1400 and 1450 km and the exobase temperature as being between 149 and 162.3 K, and Cui et al. (2009) calculate a height of ~ 1500 km and a temperature of 151 K.

In our model, we assign to each particle an initial velocity equal to the product of the most probable speed and a Gauss derivative. The most probable speed is given as the maximum value of the velocity probability density function

$$F(v) = 4\pi\left(\frac{m}{2\pi k_B T_{exo}}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T_{exo}}\right), \tag{1}$$

and is equal to

$$v_p = \sqrt{\frac{2k_B T_{exo}}{m}}, \tag{2}$$

where k_B is the Boltzmann constant, T_{exo} is the exobase temperature of the gas, and m is the particle's mass. Each particle's initial velocity is split up into three components (v_x, v_y, v_z) according to the release angles, which are uniformly distributed over 2π (see Wurz and Lammer, 2003; Wurz et al., 2007). The two components perpendicular to the radial component are combined to one vector called the tangential velocity ($v_{tan} = \sqrt{v_x^2 + v_y^2}$) while the third component corresponds to the radial velocity ($v_{rad} = v_z$).

Once a particle has obtained two initial velocity components (tangential and radial velocity), its collision-free keplerian trajectory is calculated. To implement the keplerian trajectories, we applied an eighth order f - and g-series algorithm. Let $\mathbf{r}_0 = \mathbf{r}(t_0)$ and $\mathbf{v}_0 = \mathbf{v}(t_0)$ be the particle's position and velocity vectors at time t_0. Then, $\mathbf{r}(t)$ and $\mathbf{v}(t)$, the position and velocity vectors at time t, can be written as

$$\mathbf{r}(t) = f\mathbf{r}_0 + g\dot{\mathbf{r}}_0, \tag{3}$$

$$\mathbf{v}(t) = \dot{f}\mathbf{r}_0 + \dot{g}\dot{\mathbf{r}}_0, \tag{4}$$

where f and g are the coefficient series, and $\dot{f}$ and $\dot{g}$ are their derivatives with respect to time:

$$f = 1 - \frac{u}{2}(t-t_0)^2 + \frac{uz}{2}(t-t_0)^3 + \cdots, \tag{5}$$

$$g = t - \frac{u}{6}(t-t_0)^3 + \frac{uz}{4}(t-t_0)^4 + \cdots, \tag{6}$$

where $\mu = GM$, $u = \mu r_0^{-3}$, $z = \mathbf{r}_0\dot{\mathbf{r}}_0 r_0^{-2}$ and where G is the gravitational constant and M is Titan's mass.

In our case, where we want to compute the collision probability as a function of height, we do not use constant time steps but constant altitude steps. At each point in our simulation, we take the particle's current velocity and compute the time needed to travel the length of the altitude step. This required time is then used as our next time step. The step size of the simulation is constant at 1 km which is negligible compared to the scale height and mean free path of N_2 and CH_4 at Titan's exobase. At each altitude step, the arc-length traveled and the radial and the tangential velocities are stored.

Having computed all 10^6 methane and 10^6 nitrogen molecules' trajectories, we also used the Monte Carlo model to model H, H_2, HCN and the hydrocarbons present in Titan's exosphere. These particles were not further investigated, but used, together with the methane and nitrogen molecules, to compute the total vertical density profile in Titan's exosphere $n(r)$. Each species' vertical density profile is computed separately by multiplying the species' exobase density with the fraction of particles present at each altitude step. The total vertical density profile is equal to the sum of all species' vertical density profiles. Using this total vertical density profile, we can compute the collision probability of a particle moving upward to collide with the ambient atmospheric molecules along an element of its trajectory starting at the radial distance r_j and ending at the distance r_{j+1}:

$$P_i(r_{j,j+1}) = 1 - e^{-\int_{r_j}^{r_{j+1}} n(r)\sigma_i \, d\mathbf{s}}, \tag{7}$$

where i denotes the species, r_j and r_{j+1} are the positions of the particle at the beginning and the end of the altitude step, respectively, $n(r)$ is the height dependent total atmospheric density, σ_i is the particle's cross-section and $d\mathbf{s}$ is the line segment along the particle's path. We set σ_i equal to 3.91×10^{-15} cm^2 for N_2 and 4.75×10^{-15} cm^2 for CH_4 (Chernyi and Losev, 2007).

As we mentioned, the step size is small compared to the particle's mean free path, thus we can assume $n(r)$ to be constant within one altitude step. The probability of a particle within the altitude step $[j,j+1]$ to experience a collision can therefore be simplified to

$$P_i(r_{j,j+1}) = 1 - e^{-n(r_j)\sigma_i \Delta s_j}, \tag{8}$$

where $n(r_j)$ is the atmospheric density at r_j, and Δs_j is the average distance traveled (arc-length) within the given altitude range.

We also compute the total probability for an upward moving particle to collide along its way from its current position in the exosphere r to the upper boundary of our model r_{UB}, above which we assume no collisions. For this we calculate first the total number of collisions along the particle trajectory, that is along its path over the altitude interval $[r_j, r_N]$, which is given by the integral

$$N_{i_{col}}(r) = \int_r^{r_{ub}} n(r')\sigma_i \, ds', \tag{9}$$

where $n(r')$ is the altitude dependent total density of the atmosphere, σ_i is the particle's cross-section and ds' is the arc-length corresponding to the infinitesimal height change dr'. Substituting integration with discrete summation, we get for calculation of the total number of collisions that the particle undergoes on its way