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Least squares reconstruction of non-linear RF phase encoded MR data

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ABSTRACT

Purpose: The numerical feasibility of reconstructing MRI signals generated by RF coils that produce B_1 fields with a non-linearly varying spatial phase is explored.
Theory: A global linear spatial phase variation of B_1 is difficult to produce from current confined to RF coils. Here we use regularized least squares inversion, in place of the usual Fourier transform, to reconstruct signals generated in B_1 fields with non-linear phase variation.
Methods: RF encoded signals were simulated for three RF coil configurations: ideal linear, parallel conductors and, circular coil pairs. The simulated signals were reconstructed by Fourier transform and by regularized least squares.
Results: The Fourier reconstruction of simulated RF encoded signals from the parallel conductor coil set showed minor distortions over the reconstruction of signals from the ideal linear coil set but the Fourier reconstruction of signals from the circular coil set produced severe geometric distortion. Least squares inversion in all cases produced reconstruction errors comparable to the Fourier reconstruction of the simulated signal from the ideal linear coil set.
Conclusion: MRI signals encoded in B_1 fields with non-linearly varying spatial phase may be accurately reconstructed using regularized least squares thus pointing the way to the use of simple RF coil designs for RF encoded MRI.

1. Introduction

In conventional MRI, image information is encoded in the NMR signal through the use of B_0 magnetic field gradients. Those magnetic field gradients are engineered to be linear over a suitable FOV in the magnet bore with the result that the image can be reconstructed from the NMR signal through the use of Fourier transform techniques. Non-linear magnetic field gradients have been introduced in an effort to achieve faster imaging times without inducing painful nerve stimulation. These spatial encoding magnetic field (SEM) methods include the parallel imaging technique using local gradients (PatLoc) approach [1,2], the O-Space approach [3] and, methods that use combined linear and non-linear approaches [4–6]. In the extreme, specially engineered electromagnetic B_0 gradient coils are dispensed with entirely and the native B_0 profile of a given non-homogeneous magnet is used for SEM in portable MRI designs [7]. All of these new approaches require a modification of the Fourier transform reconstruction approach [8,9] or alternative approaches with generalized projections [10,11]. In these more generalized circumstances, a least squares approach is one of the more straight-forward ones useful for image reconstruction [7].

As an alternative to pure SEM encoding, the RF encoding method also dispenses with electromagnetic B_0 gradient coils and relies on RF phase encoding within a uniform B_0 field. The original approach to RF encoding in MRI, TRansmit Array Spatial Encoding (TRASE), encodes spatial information through the use of a linear spatially varying B_1 transmit phase [12]. With RF encoding, it is possible to eliminate the traditional B_0 gradient coil systems and their associated power supply, air and water cooling systems (for themselves and their power electronics), filters and cables. By eliminating the gradient coil subsystems, a significant fraction of MRI costs related to installation, operation and maintenance can be eliminated [13]. Also, the loud acoustic noises and vibrations generated by switching gradient pulses, which may cause trouble for some patients, especially pediatric patients [12,14], are not present with RF encoding.

A linear TRASE coil creates a B_1 field with a uniform magnitude and linearly varying spatial phase. The imaging data generated with a linear coil may be reconstructed, generally after data reordering, via a Fourier transform [15,16] similar to data acquired using B_0 gradients. Actual TRASE coils create the desired B_1 field only over a limited spatial region [17], after some optimization of coil geometry, because a global B_1 field with a uniform magnitude and linearly varying spatial phase is not

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possible with currents confined to an RF coil. The design and manufacture of RF coils intended for RF encoding would be simplified if it were not necessary for the phase of the transmitted B_1 field to vary linearly. In this paper we explore the feasibility of reconstructing non-linear RF encoded signals from a mathematical perspective.

Considered mathematically, the MRI signal dataset is the result of a signal function from the image domain, here considered as a discrete lattice, to a signal range which in the case of RF encoding may be parametrized by echo number and echo train number. (RF encoded data consist of a number of separate echo trains, with a T_R spacing between them.) In the case of linear B_1 spatial phase variation, as with linear B_0 gradients, the signal function is linear (a Fourier transform). If the signal function is non-linear it is still, in principle, invertible as long as the map is one-to-one. However, the inversion problem may be ill-posed or close to ill-posed, especially when noise or non-uniform B_0 fields are present. Ill-posed inversion problems may be handled to a certain extent by regularization methods [18] such as the Tikhonov [19] or maximum entropy methods [20,21]. These regularization methods are examples of constrained least squares methods and they require the selection of a suitable regularization parameter [22–24]. Here we show that least squares inversion [25–27] without regularization may be used to reconstruct noiseless RF encoded data obtained using simple, realistic, non-linear RF encoding coils. The inversion is, however, sensitive to noise so we show that Tikhonov regularization can be used to obtain a stable solution.

2. Theory

2.1. RF spatial phase gradient configurations

A linear RF phase gradient coil produces a transverse RF B_1 field of constant magnitude and a linearly varying spatial phase. With B_0 in the z direction, $\vec{B}_0 = B_0\vec{k}$, and with $\vec{i}, \vec{j}, \vec{k}$ being unit vectors in the x, y and z directions respectively, linear RF phase gradient coils produce the following fields:

$$\vec{B}_1 = \left\|\vec{B}_1\right\|\left(\cos(G_x x+\phi_0)\vec{i} + \sin(G_x x+\phi_0)\vec{j}\right) - x \text{ direction encoding field} \tag{1}$$

$$\vec{B}_1 = \left\|\vec{B}_1\right\|\left(\cos\left(G_y y+\phi_0\right)\vec{i} + \sin\left(G_y y+\phi_0\right)\vec{j}\right) - y \text{ direction encoding field.} \tag{2}$$

$$\vec{B}_1 = \left\|\vec{B}_1\right\|\left(\cos(G_z z+\phi_0)\vec{i} + \sin(G_z z+\phi_0)\vec{j}\right) - z \text{ direction encoding field} \tag{3}$$

Where $\|\vec{B}_1\|$ is constant. Here we assume that we are working with an excited slice (the z direction encoding field may be used for slice selection, for example) and focus on encoding and reconstructing an image in the $z = 0$ x–y plane using the x and y encoding fields. There are various ways to create a spatially varying RF phase. Here, three different RF coil configurations are explored: (a) A "circular coil set" composed of a combination of circular Maxwell and Helmholtz coils (Fig. 1); (b) A "parallel conductor coil set" that follows an optimized TRASE coil as designed by Deng et al. (Fig.2) [13] and; (c) A "linear coil set" which can create the linear RF encoding B_1 fields of Eqs. (1) and (2). Eqs. (1)–(3) can only be satisfied approximately in a limited field of view using currents in essentially one dimensional RF coils, so the linear coil set represents an ideal situation.

2.2. RF encoding pulse sequence

The basic RF encoding sequence, as we use it here, consists of a 90° RF excitation pulse from a uniform coil (no spatial RF phase variation) followed by a train of 180° pulses made through appropriate RF encoding coils (having spatial RF phase variation). Between the 180° pulses,

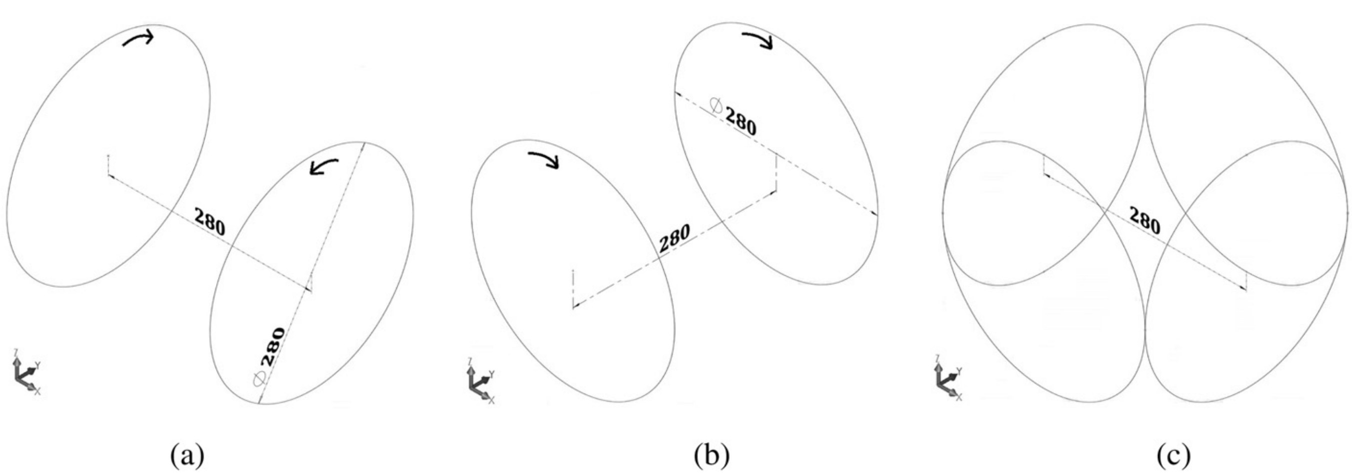

Fig. 1. The circular coil set. A pair of Maxwell (anti-parallel currents) and Helmholtz (parallel currents) coils, 28 cm in diameter, produce spatial variation of the B_1 phase. The Maxwell coils provide approximations of the sine terms in Eqs. (1) and (2) while the Helmholtz coils provide approximations for the cosine terms. For producing an RF phase gradient in x direction, the Maxwell coil axis should be placed in x direction and the Helmholtz coil axis should be placed in the y direction. An RF phase gradient in the y direction can be obtained by switching the axis of the Maxwell and Helmholtz coils. This switch may be done by changing the currents in the coils; a physical switch is not necessary. The current magnitude in the Helmholtz and Maxwell coils were set to be equal for the simulations considered here. (a) Maxwell coil for the x direction. (b) Helmholtz coil for the y direction. (c) Complete circular coil set. Two physical loop pairs are adequate to provide both x and y direction encoding. Dimensions in mm are shown.