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A statistical method for characterizing the noise in nonlinearly reconstructed images from undersampled MR data: The POCS example ☆

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ABSTRACT

The projection-onto-convex-sets (POCS) algorithm is a powerful tool for reconstructing high-resolution images from undersampled **k**-space data. It is a nonlinear iterative method that attempts to estimate values for missing data. The convergence of the algorithm and its other deterministic properties are well established, but relatively little is known about how noise in the source data influences noise in the final reconstructed image. In this paper, we present an experimental treatment of the statistical properties in POCS and investigate 12 stochastic models for its noise distribution beside its nonlinear point spread functions. Statistical results show that as the ratio of the missing **k**-space data increases, the noise distribution in POCS images is no longer Rayleigh as with conventional linear Fourier reconstruction. Instead, the probability density function for the noise is well approximated by a lognormal distribution. For small missing data ratios, however, the noise remains Rayleigh distributed. Preliminary results show that in the presence of noise, POCS images are often dominated by POCS-enhanced noise rather than POCS-induced artifacts. Implicit in this work is the presentation of a general statistical method that can be used to assess the noise properties in other nonlinear reconstruction algorithms.

1. Introduction

Undersampling of **k**-space data has been widely used as an effective approach for rapid 2D and 3D MR imaging in a variety of clinical applications [1–13]. Typically, the central zone of **k**-space is fully sampled but the peripheral zone is partially (randomly or deterministically, temporally or spatially) sampled; hence the total scan time is noticeably reduced. Reconstruction from these undersampled MR data may use the conventional linear inverse Fourier transform (iFT) approach with zero-filling (ZF) [5,12], but this generally results in low-resolution images and poor image quality, especially when it is applied to sparsely sampled (vastly undersampled) MR data necessary for ultra-fast imaging. Alternatively, the reconstruction may attempt to estimate the missing data [3,7,14,15] to obtain higher-quality images, for example in parallel imaging when using multi-coil acquisitions with specific undersampling **k**-space patterns [6]. In this regard, a variety of nonlinear reconstruction techniques [7,10,11,13,14,16–18] have also been successful in reconstructing high-resolution images from partially sampled data. Such techniques [16,19,20] are still emerging, as the demand for shorter scan times with little compromise in image quality always increases. Although the convergence and other deterministic properties of these nonlinear methods are well established, little is known about how noise in the source data influences noise in the final reconstructed image. Also, characterizing and determining the performance of any of these methods require a detailed understanding of the statistical properties of the images.

For MR magnitude images produced by linear reconstruction algorithms, such as the conventional iFT or linearly combined multi-coil data in parallel imaging, it is straightforward to characterize the noise as having a Rayleigh distribution due to the linear property of the FT and the reconstruction algorithm [21–25]. This distribution becomes Rician in regions inside the imaged object (i.e., noise + signal). Dietrich et al. [25] have shown that the noise characteristics in linearly reconstructed multi-coil images (regardless of reconstruction algorithm used) follow the Rayleigh distribution. A common and simple nonlinear multi-coil reconstruction method, i.e., the sum-of-squares reconstruction, was also investigated, in which noise

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distribution of the magnitude image was not surprisingly found to follow a non-central chi-distribution. Up to now, however, there have been limited descriptions of the noise statistics (and its related effects) when using nonlinear MR reconstruction methods [17,20,25–27]. In evaluating the noise properties of these techniques and comparing them to those obtained in other reconstruction methods (e.g., via signal-to-noise ratio, SNR, and contrast-to-noise ratio, CNR, calculations), care is warranted because the nonlinearity of the algorithm can modify the underlying noise properties of the image. Therefore, direct measurement of signal-to-noise intensity ratios [21,22] in such images can result in incorrect quantitative information and may lead to misinterpretation of image reconstruction performance. Also, reporting correct SNR efficiency measures (SNR per square root of scan time) for each imaging method becomes more important in such cases because both the noise power and the total scan time of undersampled data are considered in the calculations.

In this study, we investigate the noise properties of the projection-onto-convex-sets (POCS) MR reconstruction method with phase and data-consistency constraints. The POCS algorithm is an iterative constrained approach [7,17,18] that nonlinearly reconstructs images—usually from truncated **k**-space data—by attempting to estimate values for the missing data [18]. The POCS reconstruction algorithm has also been used for both interpolation and extrapolation of sparsely sampled data [17]. Other applications of POCS algorithm have included: motion artifact reduction [28,29], ghost correction [30], image restoration [31], and in combination with parallel imaging [32,33].

Here, we experimentally and statistically characterize the properties of noise in the POCS algorithm from phantom and human MR data. Specifically, we investigate the point spread function and propose a stochastic model for the noise distribution in POCS-reconstructed magnitude images obtained from undersampled 3D **k**-space data and compare the noise properties to those in conventional iFT with ZF. We show that the probability distribution for the noise is well approximated by lognormal and Rayleigh models. Implicit in this work is the presentation of a statistical method that can be used in assessing the noise characteristics of other nonlinear reconstruction algorithms.

2. Materials and methods

2.1. Data acquisition

Fully sampled 3D **k**-space data sets were acquired on a clinical 3.0-T MR scanner (Signa VH/i; General Electric Healthcare, Waukesha, WI, USA) using a vendor-supplied quality assurance phantom, and the legs and head of two healthy volunteers. These data sets were selected because they exhibit a variety of low- and high-resolution structures. The leg data sets were used from our previous work [17]. Informed written consents, approved by our institutional review board, were obtained from the volunteers prior to scanning. A gradient-recalled echo (GRE) sequence with standard transmit/receive head coil was used to image the phantom. A balanced steady-state free precession (bSSFP) sequence with transmit/receive body coil was used to scan the legs. A fast GRE sequence with eight-channel receive-only phased-array coil was used to scan the head. This variety of pulse sequences and objects was used to test for any sequence and object dependency in the results. Table 1 shows the typical scan parameters used for the experiments.

2.2. Phantom simulations

To separate noise from artifacts in POCS-reconstructed images and to investigate spatial variability of the noise, a set of noise-free (i.e., for artifact-only analysis) and noise-added phantom simulations were conducted. Spherical objects with uniform amplitude of 1.0 were created, and independent and identically distributed complex Gaussian noise with zero mean ($\mu = 0$) and two standard deviations levels ($\sigma_1 = 0.1$ and $\sigma_2 = 0.2$) was evenly added to the complex data.

2.3. Data undersampling and image reconstruction

A commercial program (MATLAB, Version 7.4.0, R2007a; Mathworks, Natick, MA, USA) on a general-purpose workstation (Optiplex 960, quad Q9650 at 3.0 GHz, 4.0 Gb RAM running under Microsoft Windows XP Professional x64; Dell Inc., Round Rock, TX, USA) was used to emulate undersampling and to implement the POCS and conventional iFT with ZF reconstruction algorithms. From each fully sampled **k**-space data, as illustrated in Fig. 1a, only a fraction, $C + P$, of the acquired data was retained ($0 \leq C + P \leq 1$, where C and P are fractions of data at the center and the periphery of the **k**-space, respectively), corresponding to a shortened scan time of $100(C + P)$% of the total prescribed scan time. Without loss of generality, the readout direction was denoted as k_x in this study. $100C$% of the readouts, each fully sampled along k_x, was retained as the fully sampled central (k_y, k_z)-phase-encoding zone and $100P$% of the readouts was retained in the peripheral phase-encoding zone. Two general undersampling strategies, namely i) truncated **k**-space [7,18] (Fig. 1b) and ii) sparsely sampled **k**-space (Fig. 1c) [5,17] were used. For the truncated sampling, the $100P$% phase encodings were selected on one side along the central phase-encoding zone (C region) so that C and P regions, together, made a fully sampled, shifted rectangular phase-encoded zone. For sparse sampling the $100P$% phase encodings were chosen randomly and uniformly across the peripheral zone. C ranged from 0.0156 to 0.250, corresponds to a central zone with 1/8 to 1/2 of the maximum phase-encoding values along each k_y and k_z. For a specific C value, P was varied from 0.0 to $\left(0.5 + 0.5\sqrt{C}\right)^2 - C$ (for sampling pattern of Fig. 1b), and from 0.0 to $(1 - C)$ (for sampling pattern of Fig. 1c), which resulted in shortened scan times ranging from C to $\left(0.5 + 0.5\sqrt{C}\right)^2$ and from C to 1.0, respectively; where 1.0 is the normalized total scan time of the fully sampled data. These values were selected to match scan

Table 1
Typical scan parameters used for phantom and volunteer experiments.

Scan parameters	QA Phantom (GRE)	Volunteer 1—legs (bSSFP)	Volunteer 2—head (fast GRE)
TR/TE/flip angle	8.1 ms/3.2 ms/60°	8.1 ms/4.05 ms/45°	4.7 ms/1.4 ms/35°
Receiver bandwidth	±62.5 kHz	±62.5 kHz	±62.5 kHz
FOV	20 cm	20 cm	22 cm
Acquisition matrix	256 × 256 × 64	256 × 256 × 48	256 × 256 × 48
Slice thickness	1 mm	1.5 mm	2 mm
Total scan time (full **k**-space)	133 s	100 s	58 s
Coil	Quadrature head	Body	8-Ch PA head

Abbreviations: TR, repetition time; TE, echo time; FOV, field-of-view; GRE, gradient-recalled echo; bSSFP, balanced steady-state free precession; PA, phased-array.