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The SENSE-Isomorphism Theoretical Image Voxel Estimation (SENSE-ITIVE) model for reconstruction and observing statistical properties of reconstruction operators

Iain P. Bruce[a], M. Muge Karaman[a], Daniel B. Rowe[a, b,*]

[a]Department of Mathematics, Statistics, and Computer Science, Marquette University, Milwaukee, WI, USA
[b]Department of Biophysics, Medical College of Wisconsin, Milwaukee, WI, USA

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Abstract

The acquisition of sub-sampled data from an array of receiver coils has become a common means of reducing data acquisition time in MRI. Of the various techniques used in parallel MRI, SENSitivity Encoding (SENSE) is one of the most common, making use of a complex-valued weighted least squares estimation to unfold the aliased images. It was recently shown in Bruce et al. [Magn. Reson. Imag. 29(2011):1267-1287] that when the SENSE model is represented in terms of a real-valued isomorphism,it assumes a skew-symmetric covariance between receiver coils, as well as an identity covariance structure between voxels. In this manuscript, we show that not only is the skew-symmetric coil covariance unlike that of real data, but the estimated covariance structure between voxels over a time series of experimental data is not an identity matrix. As such, a new model, entitled SENSE-ITIVE, is described with both revised coil and voxel covariance structures. Both the SENSE and SENSE-ITIVE models are represented in terms of real-valued isomorphisms, allowing for a statistical analysis of reconstructed voxel means, variances, and correlations resulting from the use of different coil and voxel covariance structures used in the reconstruction processes to be conducted. It is shown through both theoretical and experimental illustrations that the miss-specification of the coil and voxel covariance structures in the SENSE model results in a lower standard deviation in each voxel of the reconstructed images, and thus an artificial increase in SNR, compared to the standard deviation and SNR of the SENSE-ITIVE model where both the coil and voxel covariances are appropriately accounted for. It is also shown that there are differences in the correlations induced by the reconstruction operations of both models, and consequently there are differences in the correlations estimated throughout the course of reconstructed time series. These differences in correlations could result in meaningful differences in interpretation of results.

Keywords: MRI; fMRI; SENSE; SENSE-ITIVE; Parallel imaging; Sub-sampled data; Aliasing; Coil covariance; Induced operator correlation

1. Introduction

The fundamental basis for image formation in magnetic resonance imaging (MRI) is the discovery that the spatial information of an object can be Fourier encoded in the resonance spectrum by a magnetic field gradient [1,2]. Fourier encoded by magnetic field gradients, the complex-valued spatial frequencies are not measured instantaneously, but rather individually in a serial fashion, resulting in a long acquisition time for the spatial frequencies for a volume of images. As such, the measurement of sub-sampled spatial frequencies with multiple receiver coils, in parallel [3], has become a popular means of reducing image acquisition time.

The basis of parallel MRI is such that an array of receiver coils are used to acquire spatial frequencies for reduced field-of-view (FOV) images concurrently, which are combined into a single full FOV image using a technique such as SENSitivity Encoding (SENSE) [4]. The SENSE model has become a very popular parallel image reconstruction technique as it does not have strict restrictions on the layout of receiver coils, and the reduction in time achieved by sub-sampling the spatial frequencies reduces the requirements on issues such as breath holding in cardiac imaging [5]. However, the advantage gained in reducing acquisition time by sub-sampling spatial frequencies has a reciprocal effect on the time and difficulty faced in reconstructing the sub-sampled data.

* Corresponding author.
E-mail address: daniel.rowe@marquette.edu (D.B. Rowe).

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When the AMMUST (A Mathematical Model for Understanding the Statistical effects) framework for analyzing reconstruction and pre-processing operators in [6] was adapted to represent the SENSE reconstruction model in terms of a real-valued isomorphism in [7], it was found that when represented in this way, that the complex-valued normal distribution assumed in the noise between receiver coils imposes a skew-symmetric coil covariance structure. However, when estimated from two different experimental data sets, it is shown that this assumption is miss-specified. It is also generally assumed in the literature that there is an identity covariance structure between the aliased voxels in each of the coil images. This assumption is also shown to be miss-specified, as there is a non-identity voxel covariance structure when estimated directly from experimental data. With both the coil and voxel covariance structures shown to be inappropriately defined when represented in terms of a real-valued isomorphism, we propose a new image reconstruction model, entitled SENSE-Image Theoretical Isomorphism Voxel Estimation (SENSE-ITIVE), which uses both a mathematically correct covariance structure between receiver coils and a non-identity voxel covariance structure, both observed in real data.

Comparisons between the SENSE and SENSE-ITIVE models are theoretically illustrated on a time series with 490 scans (TRs) of 96×96 constant circle and Shepp-Logan phantom data, generated with covariance structures between coils and voxels similar to that of experimental data. The reconstruction techniques and statistical analysis undertaken in the theoretical illustration are then carried across to experimentally acquired spherical phantom and human subject fMRI data. A real-valued isomorphism representation of the complex-valued coil covariance matrix, as well as the covariance structure between voxels, is estimated from an experimental time series of 490 scans of a spherical phantom, and an experimental time series of resting state scans with a human subject. The time series of data acquisitions in each data set are reconstructed by both models, after which a statistical analysis is performed comparing the effects that the miss-specified coil and voxel covariance structures in the SENSE model have on voxel means, covariances, and correlations. As the results of the study conducted in this manuscript are most significant in functional connectivity studies, lower resolution 96×96 images were used in both the theoretical and experimental illustrations.

The implementation of a real-valued isomorphism in [7] showed that the mapping, or “unfolding,” of aliased voxels from the receiver coil images into a single combined image induces a correlation between the aliased voxels from each “fold.” While the SENSE-ITIVE model itself does not address this issue, a comparison of the correlations induced by the SENSE and SENSE-ITIVE reconstruction operators illustrates the effects of miss-specified coil and voxel covariance structures. As presented in [7], the implementation of image smoothing amplifies the correlation induced between aliased voxels by the SENSE image reconstruction operators, and thus image smoothing by means of a Gaussian kernel is applied in both the theoretical and experimental illustrations.

2. Theory

2.1. Linear framework

The derivation in [8] that allows for the complex-valued inverse Fourier transformation to be represented in terms of a real-valued isomorphism is the basis on which a statistical analysis of the linear operations commonly performed in image reconstruction can be performed. The mathematical formalism of image reconstruction by [8] was generalized by [6] to make use of a Cartesian linear image reconstruction, and was further extended in [7] to accommodate sub-sampled data from multiple receiver coils and represent the SENSE model as a linear operator. Traditionally, the sub-sampling of data occurs by omitting lines of k-space in the Phase Encoding (PE) direction, although the framework and principle can be applied for any direction. In this Cartesian framework, the subscript y denotes the PE direction (i.e. bottom-top), while the subscript x denotes the frequency encode direction (i.e. left-right). For an acceleration factor (also commonly known as a reduction factor), A, a receiver coil would only acquire every A^{th} line of k-space in the PE direction. Thus, a sub-sampled matrix of spatial frequencies for coil l, F_{lC}, where l=[1,2,…,N_C], would be of dimensions $(p_y/A)\times p_x$. In a real-valued isomorphism, a reconstructed complex-valued aliased image from each coil, in vector form, y_l, is represented as a product of a 2D inverse Fourier transformation operator [8], Ω, with the observed sub-sampled complex-valued k-space spatial frequencies in vector form, f_l, as

$$y_l = \Omega f_l. \tag{1}$$

If F_{lC} is a $(p_y/A)\times p_x$ matrix of sub-sampled two-dimensional complex-valued spatial frequencies, then the vector of observed k-space spatial frequencies, f_l, in Eq. (1) are formed by stacking the $p_x p_y/A$ real spatial frequencies on top of the $p_x p_y/A$ imaginary spatial frequencies

$$f_l = vec\left(\mathrm{Re}\left(F_{lC}^T\right), \mathrm{Im}\left(F_{lC}^T\right)\right), \tag{2}$$

where $vec(\cdot)$ is a vectorization operator that stacks the columns of its matrix argument, $\mathrm{Re}(\cdot)$ denotes the real part, and $\mathrm{Im}(\cdot)$ denotes the imaginary part. Similarly to the observed k-space data, if the complex-valued reconstructed image is of dimensions $(p_y/A)\times p_x$, then the reconstructed image vector will consist of $p_x p_y/A$ real reconstructed image values stacked above $p_x p_y/A$ imaginary reconstructed image values, resulting in a reconstructed image vector y of dimensions $2p_x p_y/A\times 1$.

To reconstruct sub-sampled complex-valued spatial frequencies from an array of N_C receiver coils, the vectorization in Eq. (2) is applied to each of the N_C sub-sampled spatial frequency matrices, which are in turn