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Spectral properties of quantum dots influenced by a confining potential model

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ABSTRACT

We obtain the exact energy spectra and corresponding wave functions of the spherical quantum dots for any (n,l) state in the presence of a combination of pseudo-harmonic, Coulomb and linear confining potential terms within the exact analytical iteration method (EAIM). The interaction potential model under consideration is labeled as the Cornell modified-plus-harmonic (CMpH) type which is a correction form to the harmonic, Coulomb and linear confining potential terms.

1. Introduction

In the last decades, low-dimensional quantum systems have been the focus of extensive theoretical investigations as the subject of quantum dots (QDs). Many efforts have recently been done into understanding their electronic, optical and magnetic properties. The application of magnetic field is equivalent to introducing an additional confining potential which modifies the transport and optical properties of conduction-band electrons in QDs. In addition, introducing electrical field gives rise to electron redistribution that makes change to the energy of quantum states which experimentally control and modulate the intensity of optoelectronic devices [1]. The problem of the inverse-power potential, $1/r^n$, has been used on the level of both classical and quantum mechanics. Some series of inverse power potentials are applicable to the interatomic interaction in molecular physics [2,3]. The interaction in one-electron atoms, hadronic and Rydberg atoms takes into account inverse-power potentials [4]. Indeed, it has also been used for the magnetic interaction between spin-1/2 particles with one or more deep wells [5]. The analytical exact solutions of this class of inverse-power potentials $V(r)=Ar^{-4}+Br^{-3}+Cr^{-2}+Dr^{-1}$, $A>0$, were presented by Barut *et al* [6] and Özçelik *et al* [7] by making an available ansatz for the eigenfunctions. The Laurent series solutions of the Schrödinger equation for power and inverse-power potentials with two coupling constants $V(r)=Ar^2+Br^{-4}$ and three coupling constants $V(r)=Ar^2+Br^{-4}+Cr^{-6}$ are obtained [8,9].

The analytical exact iteration method (AEIM) which demands making a trial ansatz for the wave function [7] is general enough to be applicable to a large number of power and inverse-power potentials [10]. Recently, this method is applied to a class of power and inverse-power confining potentials of three coupling constants and containing harmonic oscillator, linear and Coulomb confining terms [11]. This kind of Cornell-plus-Harmonic (CpH) confining potential of the form $V(r)=ar^2+br-c/r$ is mostly used to study individual spherical quantum dots in semiconductors [12–14]. So far, such potentials containing quadratic, linear and Coulomb terms have been extensively studied [15,16].

The present work considers the following confining interaction potential consisting of a sum of pseudo-harmonic, linear and Columbic potential terms:

$$V(r)=V_{\rm H}(r)+V_{C\text{-mod}}(r)=ar^2+br-\frac{c}{r}-\frac{d}{r^2},a>0 \qquad (1)$$

where a, b, c and d are arbitrary constant parameters to be determined latter. We will refer to this potential model in (1) as a Cornell-modified plus harmonic (CMpH) potential, since the functional form has been improved by the additional $-d/r^2$ piece; besides the contribution from the additional term also alters the value of b and c [17]. Obviously, the harmonic or power-law term has not been considered by the authors of Ref. [18] as the results are expected to be similar. For the sake of comparison, we have plotted the CpH and CMpH potentials in Fig. 1 for the values of potential parameters: $a=1.0\ \text{eV fm}^{-2}$, $b=0.217\ \text{eV fm}^{-1}$,

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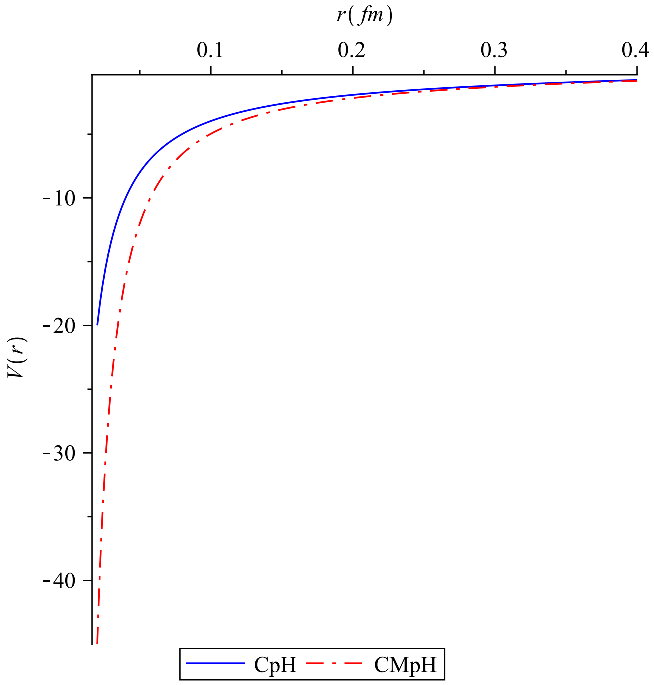

Fig. 1. A comparison between the CpH and CMpH potentials [see Eq. (1)] with the selected values of parameters: $a=1.0\ \text{eV fm}^{-2}$, $b=0.217\ \text{eV fm}^{-1}$, $c=0.400$ eV fm and $d=0.010\ \text{eV fm}^2$.

$c=0.400$ eV fm and $d=0.010\ \text{eV fm}^2$. It is seen that the CMpH potential is more singular than CpH potential when $r\to 0$.

In this work, we will apply the AEIM used in [7,11] to obtain the exact energy eigenvalues and their corresponding radial wave functions of the radial Schrödinger equation (RSE) with the CMpH potential for any arbitrary (n,l) state. Very recently, we have studied the exact analytical bound state energy eigenvalues and normalized wave functions of the non-relativistic and spinless relativistic equations with pseudo-harmonic interaction under the effect of external uniform magnetic field and Aharonov–Bohm (AB) flux field in the framework of the Nikiforov–Uvarov (NU) method [19–21].

The paper is structured as follows: In Section 2, we obtain the exact energy eigenvalues and corresponding wave functions of the RSE in three-dimensions (3D) for the confining CMpH potential model by proposing a suitable form for the wave function. In Section 3, we apply our results to an electron in spherical quantum dot of InGaAs semiconductor. The relevant conclusion is given in Section 4.

2. Exact solution of RSE with a confining potential model

The three-dimensional (3D) Schrödinger equation takes the form [22]

$$\left[-\frac{\hbar^2}{2m}\Delta+V(r)\right]\psi(r,\theta,\varphi)=E_{nl}\psi(r,\theta,\varphi) \tag{2a}$$

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{L^2(\theta,\varphi)}{\hbar^2 r^2} \tag{2b}$$

where m is the isotropic effective mass and E_{nl} is the total binding energy of the particle. The complete wave function, $\psi(r,\theta,\varphi)$in (2a) can be written as:

$$\psi(r,\theta,\varphi)=\sum_{n,l}N_l\psi_{nl}(r)Y_{lm}(\theta,\varphi) \tag{3}$$

with $Y_{lm}(\theta,\varphi)$ is the spherical harmonic part of the wave function satisfying

$$L^2(\theta,\varphi)Y_{lm}(\theta,\varphi)=\hbar^2 l(l+1)Y_{lm}(\theta,\varphi) \tag{4}$$

and the radial part of the wave function, $\psi_{nl}(r)$, is the solution of the equation

$$\left[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{l(l+1)}{r^2}+\frac{2m}{\hbar^2}(E_{nl}-V(r))\right]\psi_{nl}(r)=0 \tag{5}$$

where r stands for the relative radial coordinates. The radial wave function $\psi_{nl}(r)$ is well-behaved at the boundaries (the finiteness of the physical solution demands that $\psi_{nl}(0)=\psi_{nl}(r\to\infty)\simeq 0$). Now, employing the transformation

$$\psi_{nl}(r)=\frac{1}{r}U_{nl}(r) \tag{6}$$

reduces Eq. (5) to the simple form

$$U''_{nl}(r)+\left[\varepsilon_{nl}-a_1r^2-b_1r+\frac{c_1}{r}+\frac{d_1-l(l+1)}{r^2}\right]U_{nl}(r)=0 \tag{7}$$

where $U_{nl}(r)$ is the reduced radial wave function and we used the simplifications:

$$\varepsilon_{nl}=\frac{2m}{\hbar^2}E_{nl},\ a_1=\frac{2m}{\hbar^2}a,\ b_1=\frac{2m}{\hbar^2}b,\ c_1=\frac{2m}{\hbar^2}c,\ d_1=\frac{2m}{\hbar^2}d \tag{8}$$

The analytical exact iteration method (AEIM) requires making the following ansatze for the wave function [9],

$$U_{nl}(r)=f_n(r)\exp[g_l(r)] \tag{9}$$

with

$$f_n(r)=\begin{cases}1, & n=0,\\ \prod_{i=1}^{n}\left(r-\alpha_i^{(n)}\right) & n=1,2,\cdots,\end{cases} \tag{10a}$$

$$g_l(r)=-\frac{1}{2}\alpha r^2-\beta r+\delta\ln r,\quad \alpha>0,\beta>0 \tag{10b}$$

It is clear that $f_n(r)$ are equivalent to the Laguerre polynomials [23]. Substituting Eq. (9) into Eq. (5) we obtain

$$U''_{nl}(r)=\left[g''_l(r)+g'^2_l(r)+\frac{f''_n(r)+2f'_n(r)g'_l(r)}{f_n(r)}\right]U_{nl}(r) \tag{11}$$

and comparing Eq. (11) with its counterpart Eq. (7) yields

$$a_1r^2+b_1r-\frac{c_1}{r}+\frac{l(l+1)-d_1}{r^2}-\varepsilon_{nl}=g''_l(r)+g'^2_l(r)+\frac{f''_n(r)+2f'_n(r)g'_l(r)}{f_n(r)} \tag{12}$$

The simplest case when $n=0$ requires taking $f_0(r)$and $g_l(r)$ given in Eq. (10b) to solve Eq. (12)

$$a_1r^2+b_1r-\varepsilon_{0l}-\frac{c_1}{r}+\frac{l(l+1)-d_1}{r^2}=\alpha^2r^2+2\alpha\beta r$$
$$-\alpha[1+2(\delta+0)]+\beta^2-\frac{2\beta(\delta+0)}{r}+\frac{\delta(\delta-1)}{r^2} \tag{13}$$

Further, comparing the corresponding powers of r on both sides of Eq. (13) we find the following energy formula and the restrictions on the potential parameters as:

$$\alpha=\sqrt{a_1}, \tag{14a}$$

$$\beta=\frac{b_1}{2\sqrt{a_1}},a_1>0, \tag{14b}$$

$$c_1=2\beta(\delta+0), \tag{14c}$$