Physica B 407 (2012) 3727–3730

Contents lists available at SciVerse ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Peculiarities of propagation of acoustic excitations through an imperfect 1D-superlattice

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ARTICLE INFO

Article history:
Received 16 March 2012
Received in revised form
16 May 2012
Accepted 25 May 2012
Available online 2 June 2012

Keywords:
1D-supperlattice
Impurity layers
Virtual crystal approximation
Forbidden acoustic zone

ABSTRACT

Virtual crystal approximation is adapted to address peculiarities of propagation of acoustic waves through a 1D 'sandwich' superstructure consisting of alternating layers of two types randomly substituted by foreign layers of the third type. Same-parity layers are of the same width and constitute a sublattice. Dependence of the lowest forbidden acoustic zone width of the described structure on concentrations of impurity layers in the two sublattices is numerically evaluated for longitudinal and transverse excitations. Values of substitute concentrations making the structure completely transparent prove to be independent of the relative widths of the 1st and 2nd type layers.

1. Introduction

Practical objectives of acoustic sciences may somewhat conventionally be divided into two big tasks, namely reduction of unwanted noises and enhancement of useful acoustic signals. Numerous examples can be cited from the fields of medicine, material science, mining engineering, warfare, etc. Hence the necessity of fabricating metamaterials with predictable acoustic spectra and development of adequate theoretical models. Apart from immediate technical applications following the propagation of sound excitations is a widespread and very often indispensable method for examining solids. Acoustic techniques are being constantly improved, primarily in the direction of extension of frequency range.

Today there are substantial number of works devoted to calculation of electromagnetic and acoustic excitation spectra in superlattice systems [1–6]. For the most part they are based on the T-matrix method and involve solution of systems of equations for the Fourier components of the corresponding fields. Real acoustic superlattices are always non-ideal [7–9]. In the majority of cases that makes impossible to obtain analytically the physical characteristics of interest (such as transmission coefficients, band spectra etc.), and so the approximate and numerical methods must be employed. In Ref. [10] it is shown for instance that near the Brillouin zone optical frequencies can approximately be expressed in analytic form as functions of the Bloch wave vector.

In our previous papers we adapted the approach used for ideal superlattices [10] to study electromagnetic excitations in non-ideal 1D systems comprising randomly distributed foreign (defect) layers [5,6]. A method permitting to calculate acoustic excitation spectrum under this kind of disorder is the configuration averaging [11]. Unlike methodology of Refs. [7–9] it enables one to obtain the quantities of interest (including localized modes) as functions of impurity concentrations. Replacement of the configurationally dependent Hamiltonian parameters by their configurationally averaged values comprises the essence of the virtual crystal approximation (VCA) [12] used in Refs. [5,6] to study optical characteristics of non-ideal superlattices.

The present work concerns with extending the approach of Refs. [5,6] to investigate the peculiarities of propagation of acoustic excitations through a non-ideal phonon crystal (a system of plane-parallel homogeneous layers with differing elastic tensors). In doing so we largely rely on the ideology developed in Ref. [2]. The employed virtual crystal approximation does not allow the study of wave localization in disordered layered systems (which is done in Refs. [7–9]) but is sufficient to evaluate the forbidden acoustic zone width as a function of concentration of impurity layers, which is the purpose of this paper.

2. The model

Consider a discretely-inhomogeneous acoustic medium constituted by a set of alternating homogeneous plane-parallel layers of two types, randomly substituted by impurity layers of the third

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http://dx.doi.org/10.1016/j.physb.2012.05.051

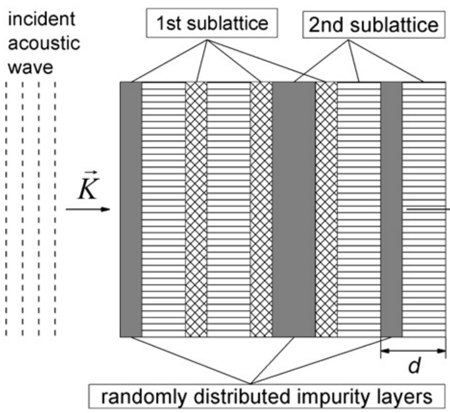

Fig. 1. Problem scheme: a two-sublattice phonon crystal with randomly included impurity layers.

type (Fig. 1). Same-parity layers have the same width and make a sublattice.

In the general case of inhomogeneous medium, whose matter density $\rho\left(\vec{r}\right)$ and elastic moduli $\hat{\Lambda}\left(\vec{r}\right)$ are the functions of coordinates of the displacement field dynamics [13] is given by the system of equations:

$$\rho(\vec{r})\ddot{u}_i(\vec{r},t) = [\hat{L}(\vec{r})]_{im} u_m(\bar{r},t), \tag{1}$$

where $\hat{L}(\vec{r})$ is a tensor operator:

$$\left[\hat{L}(\vec{r})\right]_{im} = -\frac{1}{\rho(\vec{r})}\left[\frac{\partial \Lambda_{iklm}(\vec{r})}{\partial x_k}\frac{\partial}{\partial x_l} + \Lambda_{iklm}(\vec{r})\frac{\partial^2}{\partial x_k \partial x_l}\right] \tag{2}$$

From Eq. (1) it follows that if we restrict ourselves to the case of monochromatic elastic excitations $\vec{u}(\vec{r},t) = \vec{u}(\vec{r})\exp(-i\omega t)$ the equation for amplitudes $\vec{u}(\vec{r})$ takes the form

$$\hat{L}(\vec{r})\vec{u}(\vec{r}) = \omega^2 \vec{u}(\vec{r}). \tag{3}$$

For a translation invariant system of period $\vec{d}$, operator $\hat{L}$, tensor $\hat{\Lambda}$ and density $\rho\left(\vec{r}\right)$ obey the relations

$$\hat{L}(\vec{r}) = \hat{L}(\vec{r}+\vec{d}), \quad \hat{\Lambda}(\vec{r}) = \hat{\Lambda}(\vec{r}+\vec{d}), \quad \rho(\vec{r}) = \rho(\vec{r}+\vec{d}), \tag{4}$$

and so $\rho(\vec{r})$ and $\hat{\Lambda}(\vec{r})$ can be expanded as the Fourier series in vectors of the corresponding reciprocal lattice:

$$\rho(\vec{r}) = \sum_{\vec{g}} \rho(\vec{g})\exp(i\vec{g}\cdot\vec{r}), \quad \Lambda_{iklm}(\vec{r}) = \sum_{\vec{g}} \Lambda_{iklm}(\vec{g})\exp(i\vec{g}\cdot\vec{r}). \tag{5}$$

In view of condition (4), the solutions of Eq. (3) have the Bloch form as follows:

$$\vec{u}_{\vec{K}}(\vec{r}) = \sum_{\vec{g}} \vec{U}_{\vec{K}}(\vec{g})\exp[i(\vec{K}+\vec{g})\cdot\vec{r}]. \tag{6}$$

Combining Eqs. (3), (5) and (6) yields the following system for amplitudes $\vec{U}_{\vec{K}}\left(\vec{g}\right)$:

$$\omega^2 U^i_{\vec{K}}(\vec{g}) = \sum_{\vec{g}'}\Big[B_{iklm}(\vec{g}-\vec{g}')(K^k+g'^k)(K^l+g'^l) - iA_{ilm}(\vec{g}-\vec{g}')(K^l+g'^l)\Big]\; U^m_{\vec{K}}(\vec{g}'). \tag{7}$$

Here $A_{ilm}(\vec{g})$ and $B_{iklm}(\vec{g})$ are the Fourier coefficients of functions $A_{ilm}(\vec{r}) = (1/\rho(\vec{r}))\times(\partial\Lambda_{iklm}(\vec{r})/\partial x_k)$ and $B_{iklm}(\vec{r}) = (1/\rho(\vec{r}))\Lambda_{iklm}(\vec{r})$.

Consider propagation of a monochromatic acoustic wave with the Bloch vector $\vec{K} = (o,o,K)$ in a 1D phonon crystal (z-axis is chosen to be normal to the layers). Tensor $\hat{\Lambda}(z)$ and the matter density $\rho(z)$ which depend solely on z are related to the corresponding layer-wise quantities as follows:

$$\rho(z) = \sum_{n,\alpha}\rho_{n\alpha}\theta_{n\alpha}(z), \quad \hat{\Lambda}(z) = \sum_{n,\alpha}\hat{\Lambda}_{n\alpha}\theta_{n\alpha}(z), \tag{8}$$

where

$$\theta_{n\alpha}(z) = \theta\left[z-(n-1)d-\left(\sum_{j=1}^{\alpha}a_{nj}-a_{n\alpha}\right)\right] - \theta\left[z-(n-1)d-\sum_{j=1}^{\alpha}a_{nj}\right]. \tag{9}$$

In Eqs. (8) and (9) n is the number of elementary cell in the 1D superlattice, $\alpha=1,...,\sigma$ is the number of cell element (a layer of width $a_{n\alpha}$). For an ideal 1D lattice of period d: $\rho(z)=\rho(z+d)$, $a_{n\alpha}\equiv a_\alpha$, $\rho_{n\alpha}\equiv\rho_\alpha$ (similar equalities hold for tensor $\hat{\Lambda}$). In such a case system (7) takes the form

$$\omega^2 U^i_K(g) = \sum_{g'}[B_{izzm}(g-g')(K+g')^2 - iA_{izm}(g-g')(K+g')]\;U^m_K(g'), \tag{10}$$

where $g=(2\pi/d)p$ ($p=0, \pm1, \pm2,...$). It is easy to show that in case the characteristic of layers of a 1D superlattice satisfies condition (8), tensor $A_{izm}(\vec{g})$ becomes zero. Fourier-transform of tensor $\hat{B}$, obtained with the use of expression (9), has the form

$$\hat{B}(p) = -\frac{i}{2\pi p}\sum_\alpha \frac{\hat{\Lambda}_{n\alpha}}{\rho_{n\alpha}}\left\{\exp\left(i\frac{2\pi}{d}p\sum_{j=1}^{\alpha}a_j\right) - \exp\left[i\frac{2\pi}{d}p\left(\sum_{j=1}^{\alpha}a_j - a_\alpha\right)\right]\right\}. \tag{11}$$

For isotropic layers of 1D phonon crystal tensor $\hat{\Lambda}$ (which in turn defines $\hat{B}$) is [14]

$$\Lambda_{iklm} = \lambda\delta_{ik}\delta_{lm} + \mu\;(\delta_{il}\delta_{km} + \delta_{im}\delta_{kl}), \tag{12}$$

where λ and μ are the Lamé coefficients.

Due to validity of Eq. (12), system (10) splits into two independent subsystems. One of them contains only $\Lambda_{||}=\lambda+2\mu$ (and therefore, $B_{||}$), and describes propagation of longitudinal acoustic excitations. The second one contains only $\Lambda_\perp=\mu$ (and $B_\perp$), and describes transversal excitations. Obviously, this splitting is due to layers' isotropy. In the general case of anisotropic layers the modes are coupled, which leads to development of mixed longitudinal-transverse excitations. However, for certain crystal systems and under certain orientation of crystallographic axes with respect to layers' surface [15] the modes can be uncoupled for the above form of K.

The laws of dispersion of the corresponding acoustic excitations are defined by the infinite system of Eq. (10), which for the general form of K is solved by approximate numerical methods (similar to finding polariton excitations in dielectric superlattices [1]). At the same time (as we shall see below) for K values close to the Brillouin zone boundary,

$$\left(\left|K-\frac{2\pi}{d}\right| \approx K\right)$$

the dependence $\omega=\omega(K)$ can be expressed analytically. Indeed, it can be seen from Eq. (10) that near the Brillouin zone the quantities $U^i_K(g)$ are the biggest for g with $p=0,-1$ provided that $\omega^2 \approx K^2 B_{||,\perp}(0)$ (cf. (6.1.23) in Ref. [10]). Here $B_{||,\perp}(0)\equiv B_{||,\perp}(p=0)$ is the Fourier component which is easily obtainable from (11) and (12). Keeping in system (10) only those terms corresponding to resonance of the mentioned plane waves ($p=0,-1$), we get the