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Closed form solutions for thermal stress field due to non-equilibrium heating during laser short-pulse irradiation

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Non-equilibrium heating in the lattice sub-system results in high temperature gradients in the surface region. This in turn causes thermal stress waves propagating into the substrate material. In the present study, a closed form solution for thermal stress developed in the substrate material due to volumetric pulse heating is presented. The stress free and stress continuity boundary conditions at the surface are incorporated in the closed form solutions. It is found that thermal stress wave is tensile in the surface region and it becomes compressive at some depth below the surface for stress free condition at the surface; however, it remains compressive for the condition of stress continuity at the surface.

1. Introduction

In laser short-pulse heating of metallic surfaces, electrons gain excess energy from the irradiated field and increase their temperature. This, in turn, results in thermal separation of electron and lattice sub-systems in the irradiated region. Since electron excess energy transfer to lattice site takes place through the collisional process, lattice site temperature increases gradually as the heating period progresses. The rates of electron and lattice site temperature increase change for different absorption depths and laser pulse shapes. The energy transfer in the irradiated region governs by non-equilibrium transport, which can be modeled through electron kinetic theory approach. The resulting lattice temperature equation becomes hyperbolic due to wave nature of the heating process during the short interaction time. Since the depth of absorption of the irradiated field is considerably small and heating duration is extremely short, the closed form solution of the resulting hyperbolic equation provides useful insight into the heating process. Although lattice temperature rise is gradual with time, temperature gradient remains high due to small scale of depth of absorption. This in turn, results in high stress levels in the vicinity of the irradiated surface. Consequently, the closed form solutions for temperature and thermal stress fields become essential to explore the physical aspects of laser short pulse heating of metallic substrates.

Considerable research, studies were carried out to examine non-equilibrium energy transfer in the solids due to short-pulse heating. Improved two-temperature model and its application to ultra-short laser heating of metal films were studied by Jiang and Tsai [1]. They introduced variable properties including electron heat capacity, electron relaxation time, electron conductivity, reflectivity and absorption coefficient in the analysis. Thermal behavior of their slab due to parabolic microscopic heat conduction was examined by Al-Nimr et al. [2]. They showed that the slab thermal behavior was more sensitive to the variation in electron specific heat as compared to that of lattice specific heat. High power short-pulse laser heating of a film was investigated by Saidane and Pulko [3]. They developed a transmission line matrix to account for the heat wave propagation and findings revealed that laser induced thermal waves could be used to analyze subsurface properties of the films. Laser heating of a two-layered structure and analytical formation of temperature rise were presented by Zimmer [4]. He incorporated the volumetric absorption term in the analysis and compared the analytical solution with the numerical predictions. Transient heat conduction in a two-layer axisymmetric cylindrical slab due to laser short pulse irradiation was studied by Milosevic and Raynaud [5]. They used a separation of variables technique to solve the governing heat equation. Analytical solution of hyperbolic heat equation for a finite slab was presented by Shokouhmand et al. [6]. They indicated that the hyperbolic heat conduction equation enabled to describe the highly transient temperature distribution in a finite medium accurately. Non-Fourier heat conduction model with thermal source term resembling an ultra-short high power pulsed laser heating was examined by Zhang et al. [7]. They showed that the effect of non-Fourier conduction was notable and the heat source played an

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important role in energy transport, which in turn resulted in rapid ascending of the surface temperature. Transient thermal conduction and temperature distribution generated by pulsed laser was investigated by Lu et al. [8]. They showed that the correction term due to a thermal propagation velocity, which was included in the hyperbolic equation, could be ignored for the long heating durations. Analytical solution for hyperbolic heat conduction equation in relation to laser short pulse heating was presented by Yilbas et al. [9]. Their findings revealed that internal energy gain from the irradiated field due to presence of the volumetric heat source in the hyperbolic equation resulted in rapid rise of temperature in the surface region during the early heating period. Laser short-pulse heating of gold films was studied by Yilbas and Pakdemirli [10]. They used a perturbation method to obtain closed form solution for electron and lattice temperatures. A closed form solution for Cattaneo equation due to laser irradiation pulse was presented by Al-Qahtani and Yilbas [11]. They showed that the wave nature of the heating was dominant during the period of the irradiated short-pulse; however, in the late cooling period, the wave nature was replaced by diffusional heat conduction. Al-Theeb and Yilbas [12] studied laser short-pulse heating and closed form solutions for temperature field. They demonstrated that the Laplace solution of heat equation was possible and temperature predictions obtained from the numerical method agreed well with its counterpart resulted from the analytical solution. The model selection criterion for short pulse laser heating was introduced by Hu and Dong [13]. They indicated that the surface heat source model resulted in wrong temperature distribution in the substrate material due to the lesser ratio of thermal penetration depth to the optical absorption length. A closed form solution for laser pulse heating including heating and cooling cycles was presented by Yilbas and Kalyon [14]. The findings revealed that the rate of surface temperature rise in the heating cycle and its decay rate in the coding cycle are higher for short pulses. An analytical solution for non-equilibrium heating of solid surfaces under laser irradiation pulse was obtained by Kalyon and Yilbas [15]. They showed that electron and lattice temperature became identical for the heating period beyond the thermal equilibrium time. Exact solution for temperature field due to non-equilibrium heating of solid substrate was presented by Yilbas and Al-Dweik [16]. They used Lie point symmetries to obtain lattice and electron temperatures.

Although temperature distribution inside the substrate material due to non-equilibrium heating is presented previously [16], the closed form solution for thermal stress fields left obscure, particularly for the volumetric heat source consideration. In the present study, laser short-pulse heating of solid substrate is considered and thermal stress developed due to thermal response of the irradiated region is examined. The closed form solution for thermal stress distribution inside the substrate material is presented. The analysis includes a volumetric heat source resembling the absorption of the laser irradiation pulse.

2. Mathematical analysis

The equation governing phonon temperature distribution due to short-pulse laser heating can be derived from electron kinetic theory approach, which is developed earlier [17]. The resulting heat equation for lattice site is [17]

$$\left[\left(1+\tau_s\frac{\partial}{\partial t^*}\right)-\frac{\lambda^2}{f}\frac{\partial^2}{\partial x^{*2}}\right]C_L\frac{\partial T_L}{\partial t^*}=k\frac{\partial^2 T_L}{\partial x^{*2}}+\tau_p\frac{\partial}{\partial t^*}\left(k\frac{\partial^2 T_L}{\partial x^{*2}}\right)$$
$$+\left(1+\tau_p\frac{\partial}{\partial t^*}\right)(f\,\delta I(t^*)\exp(-\delta|x^*|)) \quad (1)$$

where τ_s is the electron–phonon characteristic time ($\tau_s=C_E/G$), G is the electron–phonon coupling factor, λ is the mean free path of the electrons, f is the fraction of excess energy change, C_L and C_E are the lattice and electron heat capacities, respectively, k is the thermal conductivity, τ_p is the electron mean free time between electron–phonon coupling, $I(t)=I_0\exp(-dt)$ where I_0 is the laser peak power intensity, $\exp(-dt)$ is the temporal distribution function of laser pulse, and δ is the absorption coefficient. x^* is the lattice depth and t^* is the time variable. T_L and T_E are the lattice site and electron temperatures, respectively. Introducing the following equalities and dimensionless variables

$$\lambda^2=\frac{fk}{G},\quad \tau_s=\frac{C_E}{G}$$
$$\theta_E=\frac{T_E}{T_\circ},\quad \theta_L=\frac{T_L}{T_\circ},\quad x=x^*\delta,\quad t=\frac{t^*}{C_E/G},$$
$$\alpha=\frac{k\delta^2}{G},\quad \epsilon=\frac{C_E}{C_L} \quad (2)$$

to Eq. (1) yields finally

$$\alpha(1+\mu)\frac{\partial^3\theta_L}{\partial x^2\,\partial t}+\epsilon\alpha\frac{\partial^2\theta_L}{\partial x^2}-\frac{\partial^2\theta_L}{\partial t^2}-\frac{\partial\theta_L}{\partial t}-\beta\exp(-x)\exp(-\gamma t)=0 \quad (3)$$

where

$$\mu=\frac{\tau_p G}{C_L},\quad \beta=\frac{fI_0\delta(\mu\gamma-\epsilon)}{T_0 G},\quad \gamma=\frac{dC_E}{G}. \quad (4)$$

This model is the improved energy transport equation including ballistic effects with volumetric source in dimensionless form. Once the lattice site temperature is determined, the electron temperature can be found from [17]:

$$\frac{\partial\theta_L}{\partial t}=\epsilon(\theta_E-\theta_L) \quad (5)$$

Therefore, the energy transport equations for lattice site and electrons are

$$\alpha(1+\mu)\frac{\partial^3\theta_L}{\partial x^2\,\partial t}+\epsilon\alpha\frac{\partial^2\theta_L}{\partial x^2}-\frac{\partial^2\theta_L}{\partial t^2}-\frac{\partial\theta_L}{\partial t}-\beta\exp(-x)\exp(-\gamma t)=0,$$
$$\frac{\partial\theta_L}{\partial t}=\epsilon(\theta_E-\theta_L) \quad (6)$$

Now, assume a semi infinite substrate material heated. The boundary conditions for the problem can be written as follows:

$$\frac{\partial\theta_E}{\partial x}(0,t)=0,\quad \frac{\partial\theta_E}{\partial x}(\infty,t)=0$$
$$\theta_E(x,\infty)=\theta_0,\quad \theta_L(x,\infty)=\theta_0 \quad (7)$$

Recently, [16] constructed the closed form solutions for temperature distribution with volumetric heat sources. The solution of the boundary value problem can be given, when

$$\left\{\gamma>1 \text{ and } \gamma<\frac{\epsilon}{1+\mu}\right\} \quad \text{or} \quad \left\{\gamma<1 \text{ and } \gamma>\frac{\epsilon}{1+\mu}\right\} \text{ as follow :}$$

$$\theta_L(x,t)=\theta_0+\frac{\epsilon}{\omega\gamma}H\exp(-\gamma(t+\omega x))-\epsilon H\exp(-x-\gamma t)$$
$$\theta_E(x,t)=\theta_0+\frac{(\epsilon-\gamma)}{\omega\gamma}H\exp(-\gamma(t+\omega x))-(\epsilon-\gamma)H\exp(-x-\gamma t) \quad (8)$$

where

$$\omega=\sqrt{\frac{1-\gamma}{\alpha\gamma(\gamma(1+\mu)-\epsilon)}} \quad \text{and} \quad H=\frac{\beta}{\epsilon(\gamma^2+\gamma(\alpha(\mu+1)-1)-\alpha\epsilon)}.$$

It should be noted that the solution for $\theta_L(x,t)$ and $\theta_E(x,t)$ in Eq. (8) is only valid for $\{\gamma>1$ and $\gamma<\epsilon/1+\mu\}$ or $\{\gamma<1$ and $\gamma>\epsilon/1+\mu\}$. In which case, ε is of order 10^{-2} for metals and $\gamma<1$ for short pulse heating situation.