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Band structures and localization properties of aperiodic layered phononic crystals

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ABSTRACT

The band structures and localization properties of in-plane elastic waves with coupling of longitudinal and transverse modes oblique propagating in aperiodic phononic crystals based on Thue–Morse and Rudin–Shapiro sequences are studied. Using transfer matrix method, the concept of the localization factor is introduced and the correctness is testified through the Rytov dispersion relation. For comparison, the perfect periodic structure and the quasi-periodic Fibonacci system are also considered. In addition, the influences of the random disorder, local resonance, translational and/or mirror symmetries on the band structures of the aperiodic phononic crystals are analyzed in this paper.

1. Introduction

Since the discovery of quasicrystals by Shechtman et al. [1] in 1984, aperiodic structures have been extensively studied and many results have been obtained for electromagnetic waves (EW) propagation in the so-called "aperiodic photonic crystals" [2–7]. In addition to the investigation of the electronic and optical properties of these aperiodic systems, there is recently a growth interest to study the elastic/acoustic properties of the "phononic crystals" (PNCs), with various stacking order such as the periodic, defected and random one been investigated both theoretically and experimentally [8–13]. Compared to the periodic and random phononic crystals, aperiodic phononic crystals (APNCs) share distinctive physical properties with both periodic media, i.e. the formation of well-defined band gaps, and disordered random media, i.e. the presence of localized eigenstates, thus offering an almost unexplored potential for the control and manipulation of localized field states. Accordingly, the theoretical understanding of the complex mechanisms governing elastic band gaps and mode formation in aperiodic structures becomes increasingly more important. Albuquerque [14] considered acoustic wave propagation in a solid/liquid phononic Fibonacci structure. Aynaou et al. [15] studied the propagation and localization of acoustic waves in Fibonacci phononic circuits. Fang et al. [16] discussed acoustic wave propagation in a quasi-periodic layered spherical structure. In all of these studies, only longitudinal wave characterized by transmission spectrum was considered by assuming the wave propagation direction normal to layers. Recently, Chen et al. [17] have used the well-defined localization factor to examine the behaviors of elastic waves propagating obliquely in a one-dimensional phononic quasicrystal. Coupling of longitudinal and transversal elastic waves is involved in this case. However, to the best of our knowledge, a rigorous investigation of the band gaps and localization in more complex types of aperiodic structures has not been reported so far.

In this paper we will discuss the band structures and localization properties of aperiodic phononic crystals based on Thue–Morse and Rudin–Shapiro sequences, which are less frequently considered. In particular Rudin–Shapiro sequences exhibit peculiar characteristics and are not covered by the theorems valid for other sequences. The well-defined localization factor [18] is introduced to study elastic wave propagation in these aperiodic systems instead of calculating the transmission coefficients as did in the previous publications [15,16]. The results show some merits of the localization factor calculated using the transfer-matrix method. For comparison, the band structures of the crystals with perfect periodicity and Fibonacci sequence are also discussed using the concept of the localization factor. The results show that more band structures and wider band gaps generate when aperiodicity is introduced to the phononic crystals. This may be of practical importance not only because one expect to tune the band structures, but also because we can thus control the

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propagation behavior of elastic waves by intentionally introducing the aperiodicity. In addition, the influences of the random disorder, local resonance, translational and/or mirror symmetries on the band structures of the APNCs are analyzed in this paper.

2. Theoretical method

The APNCs considered here are arranged as Thue–Morse (TM) and Rudin–Shapiro (RS) sequences. These aperiodic structures can be generated by their inflation rules, as follows: $A \to AB$, $B \to BA$ (TM) and $AA \to AAAB$, $AB \to AABA$, $BA \to BBAB$, $BB \to BBBA$ (RS), where A and B are sub-layers made up of different materials. Suppose the thickness, Lamé constant, shearing modulus and mass density of two different layers are α_j, λ_j, μ_j and ρ_j, respectively, where the subscript $j=1$ refers to material A and $j=2$ to material B.

Let us consider an in-plane wave with coupling of longitudinal and transverse modes oblique propagating in an arbitrary direction ($0^\circ \le \theta \le 90^\circ$) in the above layered systems. For convenience, we introduce two scalar potentials, φ for the longitudinal mode and ψ for the transversal mode [19], which satisfy the following equations:

$$\nabla^2 \varphi = c_L^{-2} \ddot{\varphi}, \nabla^2 \psi = c_T^{-2} \ddot{\psi}, \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$; $c_L = \sqrt{(\lambda+2\mu)/\rho}$ and $c_T = \sqrt{\mu/\rho}$ are the longitudinal and shear wave speeds, respectively. Introducing the dimensionless local coordinates:

$$\xi_j = x_j/\overline{a}_1, \eta_j = y_j/\overline{a}_1, \tag{2}$$

where $\overline{a}_1$ is the mean value of the thickness of material A. Considering Eq. (2) and the Snell's Law, we have the general dimensionless solutions to Eq. (1):

$$\begin{aligned}\varphi_j(\xi_j, \eta_j, t) &= [A_1 \exp(-iq_{Lj}\xi_j) + A_2 \exp(iq_{Lj}\xi_j)]\exp(ik_y\eta_j - i\omega t),\\ \psi_j(\xi_j, \eta_j, t) &= [B_1 \exp(-iq_{Tj}\xi_j) + B_2 \exp(iq_{Tj}\xi_j)]\exp(ik_y\eta_j - i\omega t)\end{aligned} \tag{3}$$

where $0 \le \xi_j \le \zeta_j = a_j/\overline{a}_1$; $i^2 = -1$; ω is the circular frequency; k_y is the dimensionless wave vector component along the y-axis; $q_{Lj} = \sqrt{(\omega\overline{a}_1/c_{Lj})^2 - k_y^2}$, $q_{Tj} = \sqrt{(\omega\overline{a}_1/c_{Tj})^2 - k_y^2}$; A_1, A_2, B_1 and B_2 are the unknown coefficients to be determined. For forming a state vector in the following analysis, the dimensionless displacement and stress components are given by

$$\begin{aligned}&\overline{v}_x = \frac{\partial\varphi}{\partial\xi} + \frac{\partial\psi}{\partial\eta}, \overline{v}_y = \frac{\partial\varphi}{\partial\eta} - \frac{\partial\psi}{\partial\xi},\\ &\overline{\sigma}_x = \lambda\left(\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\varphi}{\partial\eta^2}\right) + 2\mu\left(\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\xi\partial\eta}\right), \overline{\tau}_{yx} = \mu\left(2\frac{\partial^2\varphi}{\partial\xi\partial\eta} + \frac{\partial^2\psi}{\partial\eta^2} - \frac{\partial^2\psi}{\partial\xi^2}\right),\end{aligned} \tag{4}$$

We take the non-dimensional state vectors at the left and right sides of each layer (A or B) in the kth unit cell as $\boldsymbol{V}_{Lj}^{(k)} = \{\overline{\sigma}_{xLj}^{(k)}, \overline{\tau}_{yxLj}^{(k)}, \overline{v}_{xLj}^{(k)}, \overline{v}_{yLj}^{(k)}\}^T$ and $\boldsymbol{V}_{Rj}^{(k)} = \{\overline{\sigma}_{xRj}^{(k)}, \overline{\tau}_{yxRj}^{(k)}, \overline{v}_{xRj}^{(k)}, \overline{v}_{yRj}^{(k)}\}^T$ where the subscripts L and R denote the left and right sides of the layers, respectively. These two state vectors have the following relation:

$$\boldsymbol{V}_{jR}^{(k)} = \mathbf{T}_j' \boldsymbol{V}_{jL}^{(k)}, \tag{5}$$

where $\boldsymbol{T}_j'$ is a 4×4 transfer matrix of which the elements are shown in Ref. [20]. Applying the continuous conditions at the interfaces between the two layers and between the two unit cells, we have

$$\boldsymbol{V}_{2R}^{(k)} = \mathbf{T}_k \boldsymbol{V}_{2R}^{(k-1)}, \tag{6}$$

where $\boldsymbol{T}_k$ is the transfer matrix between two consecutive unit cells and is given by

$$\mathbf{T}_k = \mathbf{T}_2' \mathbf{T}_1'. \tag{7}$$

Then the total transfer matrix is obtained as $\mathbf{T} = \mathbf{T}_n \mathbf{T}_{n-1} \cdots \mathbf{T}_k \cdots \mathbf{T}_1$. If the dimension of the transfer matrices is $2m \times 2m$, then the smallest positive Lyapunov exponent γ_m is the localization factor. The expression was given by Wolf [21]:

$$\gamma_m = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \ln \|\hat{\mathbf{v}}_{2R,m}^{(k)}\|, \tag{8}$$

where the vector $\hat{\mathbf{v}}_{2R,m}^{(k)}$ is obtained through iteration and Gram–Schmidt orthonormalization procedures. For details, we refer to Ref. [18].

3. Numerical examples and discussions

First, we consider the in-plane wave propagating with the incident angle $\theta = 40^\circ$ along the Thue–Morse PNC, which is made up of Pb and Epoxy. For comparison, the material parameters are chosen as follows: $\rho_1 = 11.4 \times 10^3$ kg/m^3, $\rho_2 = 1.2 \times 10^3$ kg/m^3, $c_{L1} = 2160$ m/s, $c_{L2} = 2830$ m/s, $c_{T1} = 860$ m/s and $c_{T2} = 1160$ m/s, coincident with those recently demonstrated in Ref. [20]. The thickness of each layer is normalized by $\overline{a}_1 = a_1$. We take $\zeta_1 = a_1/\overline{a}_1 = 1.0$ and $\zeta_2 = a_2/\overline{a}_1 = 0.5$. For clarity, we introduce the dimensionless frequencies: $\Omega_{L1} = \omega\overline{a}_1/c_{L1}$, $\Omega_{L2} = \omega\overline{a}_1/c_{L2}$, $\Omega_{T1} = \omega\overline{a}_1/c_{T1}$ and $\Omega_{T2} = \omega\overline{a}_1/c_{T2}$. According to Eq. (8), n should be infinity to calculate the localization factor. However, in practical computation, a finite (but sufficiently large) number of n has to be selected. Here, we select $n = 128$, 512, 1024 to discuss the influence of the choice of n on accuracy of the localization factor. As shown in Fig. 1, the values of the localization factors become stable and the band gaps appear clearly with the increase of n. Thus, we take 2048 layers (i.e. the $11-order$ aperiodic sequence with $1-order=AB$) to carry out the computation. Compared with the Fibonacci sequence (see Fig. 3 in Ref. [20]), the Thue–Morse sequence exhibits more obvious band-splitting phenomenon, i.e. almost all band gaps (the frequency intervals with localization factors much bigger than zero) are split into more parts and very narrow pass bands where the intervals with localization factors being zero, (2.058, 2.18) and (9.71, 9,72), in the Fibonacci sequence vanish in the Thue–Morse sequence. Usually, the band gaps are studied by calculation of dispersion relations or transmitted wave amplitudes. However, a new parameter-localization factor is introduced in this paper to describe the band gaps. To check the validity of localization factors

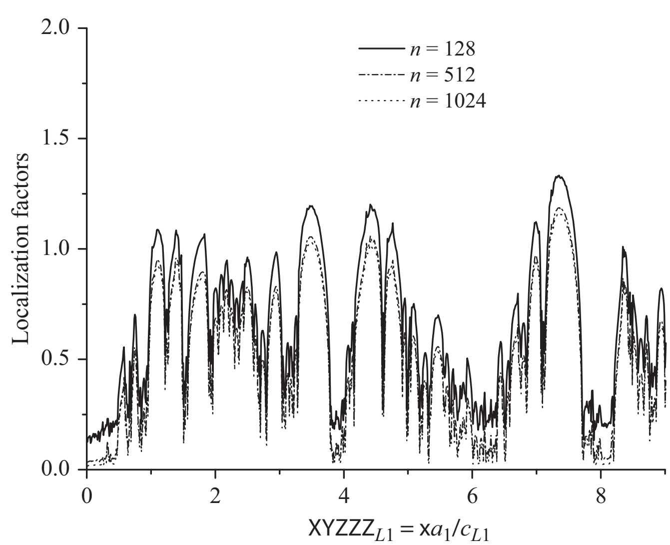

Fig. 1. Localization factors for in-plane waves propagating with the incident angle 40° in Thue–Morse phononic crystal calculated by choosing different values of n.