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Observing build up of geometric phase in an adiabatic RF flipper with the amplitude of its rotating field

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ABSTRACT

To demonstrate that adiabatic RF flippers impose an inherent geometric phase on the neutron polarization vector, we built a NSE setup consisting of two pairs of such flippers in a pulsed neutron beam. As is well known, the combined gradient and RF fields in each flipper—in the rotating frame—behave as a rotating field. The amplitude of this field in the first three flippers was kept maximum. For various amplitudes of the rotating field in the remaining flipper we measured the NSE pattern. Besides the shift of the NSE-point due to the variation of the dynamic phase, the NSE patterns show the development of the geometric phase.

1. Introduction

As is well known (see for instance Ref. [1]), each component of the spinor $|\chi\rangle$ describing a neutron flying through a magnetic field configuration characterised by $\vec{B}(t)$ (with $t{=}x/v$), picks up a phase $e^{\pm i\phi}$ where $\phi = \int_0^L |B|(x)\, dx$, the so-called "*dynamic*" phase. Since the time spent in the magnetic field is proportional to the wavelength λ, the phase ϕ is proportional to λ. When the resulting vector $\vec{B}$ of the field configuration describes a closed loop (as seen by the flying neutron), each spin state also collects a "*geometric*" phase, independent of λ, denoted ϕ_g, which is equal to the solid angle subtended by this closed loop, as shown by Berry [2].

Here we discuss an adiabatic RF/gradient flipper as described for the first time by Bazhenov et al. [3]. When such a flipper is viewed in a rotating coordinate system in the way as done long ago in NMR theory [4], the field resulting from its (DC) gradient field and RF coil can be modelled as a field of strength A rotating uniformly by π over the length l of the flipper. By reversing the gradient or the phase of the RF field, the sense of rotation can be reversed.

When the polarisation vector of the neutron beam follows this rotating field adiabatically, it has picked up a geometric phase 2π at the end of the flipper. This is an intrinsic property of such a flipper. The fact that the collected phase contains a geometric term, was demonstrated in our earlier paper [5] by comparing the phases upon reversing the gradient in the flipper and by inserting a 2 π–twisted coil between two flippers.

The purpose of this paper is to revisit the phase collected in one flipper as a function of the strength A (or the adiabaticity parameter $k \equiv \omega_L/\omega_g$ where $\omega_L = \gamma A$ is the Larmor frequency and $\omega_g = \pi(v/l)$ the geometric frequency as experienced by the flying neutron). In Ref. [5] we derived in first order of the wavelength λ of the neutron beam that after a configuration of two flippers we obtain an extra phase with a deviating dependence on λ, superimposed on the dynamic phase. Here we give exact results for this extra phase together with the limiting case $\lambda \to 0$.

The phase collected by the polarisation in one flipper cannot be directly observed, because, seen in the laboratory system, after the flipper the polarisation vector rotates at twice the rate corresponding to the DC-field B_0 in the flipper [6]. After a second flipper (which will also impose a geometric phase) the polarisation is again stationary and its phase can be measured in the lab system. In practice the collected phase amounts to $10^3-10^4 \times 2\pi$, which will give full damping of the polarisation signal. To compensate this huge phase, we transmit the beam through a second set of two flippers, with reversed precession: neutron spin echo (NSE) mode. The two sets of two flippers are referred to as NSE arm 1 and arm 2.

Commonly, in a NSE experiment the degree of the resulting polarisation is measured by off-setting the NSE condition (i.e. net precession phase $\varphi_{net} = 0$) by varying the field in a so-called "phase coil" in one NSE-arm. One measures the amplitude of the damped oscillating signal (for the whole spectrum) thus obtained: the so-called "spin-echo group" (NSE group). Here we are interested in its phase.

2. Disturbance of the NSE map by geometric phase

Let the precession phase ϕ_1 in NSE arm 1 be composed of a dynamic and a geometric term: $\phi_1 = \Phi_1\lambda + \phi_{g1}$. In general, the precession $\phi_2 = \Phi_2\lambda + \phi_{g2}$ in arm 2 will not cancel ϕ_1.

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Let the dynamic phase cancel for the field value B_f in the phase coil, so $\Phi_1\lambda=(\Phi_2-c_{Ph}B_f)\lambda$ for all λ, hence $\Phi_1-\Phi_2+c_{Ph}B_f=0$ (c_{Ph} is a constant characterizing the coil, not to be further specified). When the field is varied ΔB away from B_f, a net phase $\varphi_{net}(\lambda)$ remains:

$$\varphi_{net}(\lambda)=(\Phi_1-\Phi_2-c_{Ph}(B_f+\lambda\Delta B))+\phi_{g1}-\phi_{g2}=\lambda\Delta B+\phi_{g1}-\phi_{g2}.$$

Defining the *net* geometric phase $\phi_g=\phi_{g1}-\phi_{g2}$, the measured polarisation $P\equiv\cos\varphi_{net}(\lambda)$ is

$$P(\Delta B,\lambda)=\cos(\phi_g-c_{Ph}\lambda\Delta B). \tag{1}$$

In the top row of Fig. 1 $P(\Delta B,\lambda)$ is plotted as the *NSE map* for a net (constant) geometric phase ϕ_g equal to 0, $\pi/2$, (arbitrary) 0.57 and for a (λ dependent) $\phi_g=-0.2/\lambda$. In the bottom of each panel one can see the *NSE group*: the $\lambda-$averaged polarisation (assuming a Maxwellian wavelength distribution for a source temperature 350 K). In the bottom row the same is plotted, but $P(\Delta B,\lambda)$ transformed to $P(\Delta B,1/\lambda)$. In this representation, for a pure $\lambda-$independent geometric phase the boundary lines between regions with $P>0$ and $P<0$ become straight lines. However, for an extra phase with deviating $\lambda-$dependence, these lines are curved.

We conclude that for an arbitrary geometric phase unequal zero there is no unique phase coil field where P has the same phase for all λ.

3. Precession phase in one NSE arm with different amplitudes in flippers

As discussed above, for measuring the total phase collected in one adiabatic RF/gradient flipper we arrive at a NSE setup in which each arm consists of two such flippers, denoted F1,F2/F3,F4. In fact we used an existing setup [5] installed in a pulsed "white" beam, with TOF-data collection.

In Ref. [5] we derived that for neutron velocity v the final precession phase ϕ_{ff} of the polarisation vector after one NSE arm with two adiabatic/RF flippers of length l, a distance L apart, which are synchronously operated at amplitudes A_1 and A_2 of their rotating field, is

$$\phi_{ff}(l,L,v,A_1,A_2)=\Phi_{ZF}-\Phi(l,A_1)+\Phi(l,A_2), \tag{2}$$

with

$$\Phi_{ZF}=2\omega_0(L/v)\quad\text{with } \omega_0=\gamma B_0 \text{ and} \tag{3}$$

$$\Phi(l,A_i)=\gamma\frac{l}{v}A_i\sqrt{1+\left(\frac{\pi}{\gamma(l/v)A_i}\right)^2}\quad(i=1,2). \tag{4}$$

Φ_{ZF} and $\Phi(l,A_i)$ are the phases collected between the flippers [6] and inside each, respectively. The latter do not depend on the static field B_0.

In Ref. [5] we substituted Eq. (4) into Eq. (2) and gave the result *in first order* of $1/\lambda$:

$$\phi_{ff}=\Phi_{ZF}-\gamma\frac{l}{v}(A_1-A_2)-\left(\frac{1}{\gamma(l/v)A_1}-\frac{1}{\gamma(l/v)A_2}\right) \tag{5}$$

denoted

$$\phi_{ff}=\Phi_{ZF}-[\Delta\Phi_{dyn}(A_1,A_2)-\phi_r(A_1,A_2)]. \tag{6}$$

An *exact* elaboration of Eq. (2) with Eq. (4) is given in Fig. 2. The terms $\Phi(l,A_i)$ $(i=1,2)$ as given by Eq. (4) are plotted in Fig. 2(a), (d) as a function of λ and $1/\lambda$, for flippers of length l=80 mm, with A_1=1.5 mT and A_2 as indicated. For $\lambda\to\infty$ each phase $\Phi(l,A_i)$

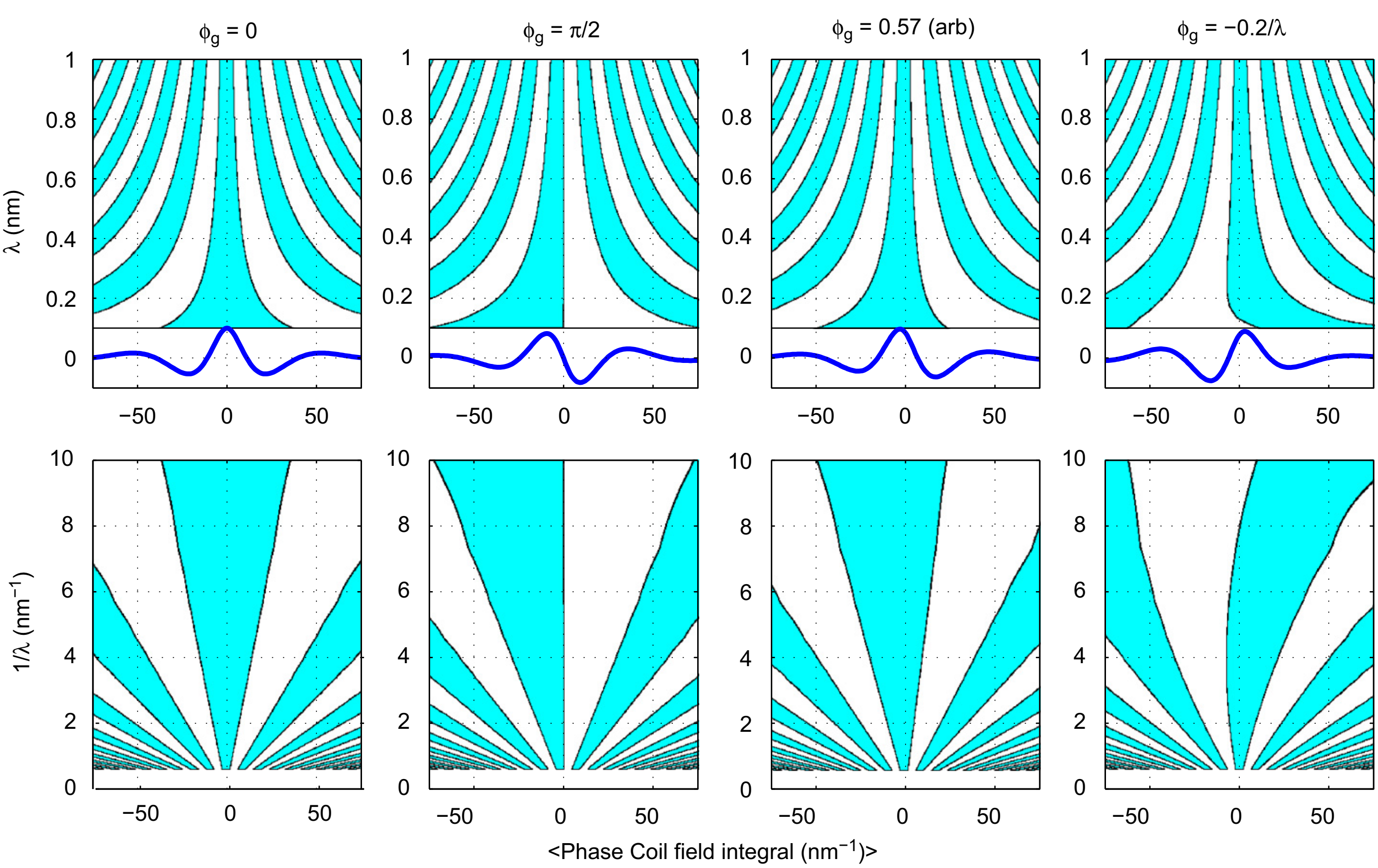

Fig. 1. Top row: Maps of the polarisation $P(\Delta B,\lambda)$ as a function of phase coil field ΔB (resolved to λ and averaged over λ) for some assumptions for the net geometric phase ϕ_g. Bottom row: the same as in top row, but λ transformed to $1/\lambda$, giving the map $P(\Delta B,1/\lambda)$. (Dark: $P>0$, white: $P<0$).