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Particle trajectories in a four rod crab cavity

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ABSTRACT

As part of the HL-LHC upgrade it is planned that crab cavities will be installed into the machine to control and increase the luminosity. Of crucial importance to the dynamics of crab cavities in the LHC machine is the modelling of the particle motion through the cavity structure, which, in some studies, has been done using thin lens kicks derived from a thin lens Hamiltonian. In this paper we study particle trajectories using field fitted vector potentials in the four rod crab cavity and make a comparison to the dynamics generated using the thin lens model.

1. Introduction

The Large Hadron Collider (LHC) is a two beam synchrotron proton collider located at CERN, Geneva. The machine is now operating successfully, delivering proton–proton luminosity to its several experiments, and the luminosity upgrade for the two high luminosity experiments ATLAS (located at interaction point 1, IP1) and CMS (located at IP5) is now being planned. The goal is a levelled instantaneous luminosity of $5\times10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$, requiring a peak instantaneous luminosity of $10\times10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ [1].

The LHC bunch spacing of 25 ns necessitates the use of a beam crossing angle at the IP to avoid additional (parasitic) collisions between the bunches, resulting in a reduction of geometric overlap and hence a reduction in luminosity. This rather inefficient way of colliding the proton bunches can be mitigated by tilting the bunches in the plane of the crossing angle, thus recovering a head-on collision geometry. Such a scheme involves an appropriate transverse kick [2] and can be achieved with a deflecting RF cavity operating in the crabbing mode, known as a crab cavity. The use of crab cavities is also a potential method to provide luminosity levelling through the fill by adjustment of the crab voltage [3].

The purpose of a crab cavity is essentially to provide a transverse kick to the protons in the bunch which varies linearly with the longitudinal bunch coordinate. The translation of the transverse kick into a beam rotation at the IP depends on the intervening beam optics and the value of the β-function at the crab cavity location. The physical limits of the cryostat lead to the requirement of a compact cavity size, with the cavity operating in a TEM mode, to allow the cavity to fit. The cavities are intended to be placed in a two beam pipe region of the long straight section on either side of the IP [4], creating a closed 'crab bump' across the IP. This is known as a local crabbing scheme.

The dynamical requirement is that the kick is linear with z (bunch position relative to synchronous particle) for a design parameter bunch length of 0.075 m [1], hence a frequency $\omega=2\pi\times400$ MHz is chosen to give a sufficient linear region in the sinusoidal momentum kick to cover the LHC bunch length, as shown in Fig. 1). This linearity is important as non-linearities could lead to increased emittance and beam losses due to non-closed crabbing bumps.

The total voltage required from two to three crab cavities before the IP is [2]

$$V_1=\frac{c^2 p_s \tan\left(\frac{\theta}{2}\right)}{q\omega\sqrt{\beta^*\beta_{\mathrm{crab}}}\sin(\Delta\phi_0)} \tag{1}$$

where the p_s is the proton momentum, β^* and β_{crab} are the values of the β-function at the IP and crab cavity, respectively, θ is the full crossing angle, q is the proton charge and $\Delta\phi_0$ is the phase advance between the cavity and the IP. This voltage is approximately 3 MV per cavity. The total voltage required to close the crab bump is

$$V_2=-R_{22}V_1 \tag{2}$$

where R_{22} is the relation between the initial and final momentum in the kick plane between the cavities. An optical constraint resulting from this is that there is no relation between the initial transverse momentum and the final position in the kick plane, requiring

$$R_{12}=0 \tag{3}$$

between the crab cavities either side of the IP.

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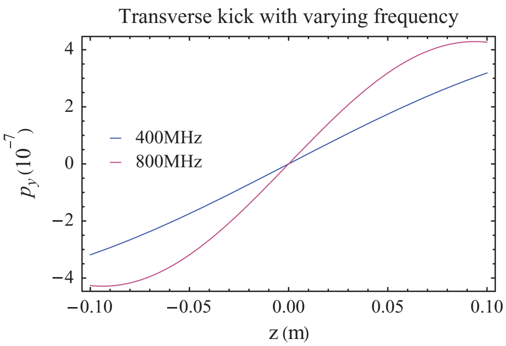

Fig. 1. Effect of frequency on magnitude of kick with kicks normalised to 3 MV from 400 MHz cavity (Eq. (6)).

The aim of this paper is to completely model the linear and non-linear beam dynamics in a crab cavity, beginning with fitting the vector potential to the field map [5] and the subsequently computing the particle trajectories using a symplectic integrator. This approach will allow modelling of the detailed particle motion in a cavity and move beyond the thin lens approximation [6], which models the crab cavity as a thin lens kick dependent on frequency and voltage [7]. In any multi-turn machine subtle dynamics may have large effects on the long term stability of a beam.

In standard notation the thin lens crab kick (for a vertically[1] crabbing cavity) can be derived from the Hamiltonian:

$$H=\frac{qV}{p_0}y\sin\left(\frac{\omega z}{c}+\Phi\right) \tag{4}$$

where H denotes the Hamiltonian, V is the voltage of the cavity, p_0 is some reference momentum, Φ is the synchronous phase of the crab cavity, y is the vertical coordinate, z is the longitudinal coordinate with respect to the bunch centre and ω is the angular RF of the cavity. The resulting kicks of this map lead to a changes in p_y and p_z, where p_y and p_z are the momenta conjugate to y and z, respectively. These kicks are easily obtained from Hamilton's equations. To model a cavity of finite length l, such as the four rod crab cavity, this thin lens kick is sandwiched between the maps for drifts of length $l/2$. By the use of a Lie transformation:

$$X\mapsto e^{-s:H:}X \tag{5}$$

the overall map for the drift-kick-drift structure can be obtained, resulting in maps for the canonical coordinates (x,p_x), (y,p_y) and (x, δ) as

$$x\mapsto x+lp_x$$

$$p_x\mapsto p_x$$

$$y\mapsto y+lp_y-\frac{l}{2}\frac{qV}{p_0}\sin\left(\frac{\omega z}{c}+\phi\right)$$

$$p_y\mapsto p_y-\frac{qV}{p_0}\sin\left(\frac{\omega z}{c}+\phi\right)$$

$$z\mapsto z$$

$$\delta\mapsto\delta-\frac{qV}{p_0}\frac{\omega y}{c}\cos\left(\frac{\omega z}{c}+\phi\right). \tag{6}$$

This gives relations between initial and final coordinates of a particle traversing the crab cavity. We refer to this model of the cavity as the thin lens, or idealised, model.

[1] The choice of transverse kick plane is arbitary for the dynamic study of a single cavity.

An alternative approach is the use of the Panofsky–Wenzel theorem [8] to compute a set of time dependent multipoles for the cavity effect on the beam. The relationship between the time dependent multipole dynamics and the approach advocated in this paper is under study.

The layout of this paper is as follows. In Section 2 we introduce the two methods used to integrate the particle trajectories through the cavity fields, standard Velocity-Verlet integration of the Lorentz Force law and symplectic integration using the split Hamiltonian method of Yoshida for a fitted vector potential. We then use these two methods to compute explicit particle trajectories through the four rod crab cavity in Section 3 and make a comparison to the thin lens model in Section 4. Finally we draw our conclusions in Section 5. Note that we confine ourselves to on-axis particles in the linear field region of the cavity in the present study, to compare the dynamics obtained with the symplectic integrator and the thin lens model.

2. Numerical integration methods

The aim of this paper is to compute particle trajectories through a thick crab cavity using a suitable Hamiltonian, fitted to the field map. To verify the numerical results we also compute the particle trajectories using the Lorentz force law:

$$\vec{F}=q(\vec{E}+\vec{v}\times\vec{B}) \tag{7}$$

where the a particle of velocity $\vec{v}$ moves through electric ($\vec{E}$) and magnetic fields ($\vec{B}$) and experiences a force $\vec{F}$. This force is applied along a path using the Velocity-Verlet algorithm. This provides a straightforward cross-check of our methods. In this section we describe both techniques.

2.1. Velocity-Verlet integration

The Velocity-Verlet algorithm is a partially symplectic integrator, which means that the energy deviation oscillates with steps rather than continuously growing as occurs with methods such as Euler and Runge–Kutta integration. The variables in this integration method are x, P_x, y, P_y, s, P_s, where P_i is the mechanical momentum.

The Lorentz force is velocity dependent so requires a predictive momentum step in addition to the standard Velocity-Verlet method [9]. Initially the position is evolved to some time Δt after the current time:

$$x(t+\Delta t)=x(t)+v(P(t))\Delta t+\frac{1}{2m\gamma(P(t))^3}F(t,x(t),P(t))\Delta t^2. \tag{8}$$

From this new position the momentum is calculated at this new time from the new position. However, since the new momentum is unknown this is only a prediction:

$$P_{pred.}(t+\Delta t)=P(t)+F(t,x(t),P(t))\Delta t. \tag{9}$$

A more precise momentum is calculated by averaging the force between the initial and final position using the predicted momentum for final Lorentz force term, such that,

$$P(t+\Delta t)=P(t)+\frac{1}{2}(F(t,x(t),P(t))+F(t,x(t+\Delta t),P_{pred.}(t+\Delta t)))\Delta t. \tag{10}$$

The γ and velocity v are calculated at each step during the integration from the particle momentum. The fields used come from an eigenmode solver and the magnitude of the complex E and B fields are taken. The fields from the solver are normalised to 1J of energy stored in the fields. The fields are then renormalised by the p_y kick given by the analytical map (Eq. (6)).