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A coincidental timing model for the scintillating fibers

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ABSTRACT

A model describing the coincidental timing of scintillating fibers is developed. Fiber geometry, the rate of scintillation decay together with the mean number, spatial dispersion and attenuation of emitted photons are considered. For a specific selection of probability distributions and parameters involved, the entire coincidental timing distributions, corresponding FWHM values and the photon detection efficiencies are extracted. The significance of the number of photons from the scintillation process is specially emphasized. Additionally, the model is extended to include a triggering feature, experimentally realized by coupling fibers to any photon resolving device. Finally, the measurements of a coincidental timing distribution were performed, with an excellent agreement found between the experimental and predicted theoretical results.

1. Introduction

Scintillating fibers have been extensively investigated from both theoretical and experimental point of view. Consequently, their scintillation mechanisms and light transmission properties are well known and conveniently summarized in Ref. [1], together with a list of the most relevant references on the subject. Nowadays, there is an active effort to include fibers as an integral part of sophisticated detector systems for nuclear experiments. Being intended for a detection of charged particles and/or electromagnetic radiation, fibers are required to provide an adequate, if not excellent coincidental timing resolution, vital for precise reconstruction of the scintillating pulse position along their length. Compared to the resolution of bulk scintillators—an order of magnitude below the nanosecond scale [2]—early measurements indicate that attainable values are well above this range [3]. The rate of scintillation process, together with a low number of detected photons being subject to a wide spatial dispersion, is considered to be the main cause for the coincidental timing discrepancy with that of bulk scintillators. Commonly, Monte Carlo simulations present a natural tool for estimation and prediction of experimental results, easily capable of including all the required physical principles. However, in this paper an analytical model is developed, describing an asymptotic form of otherwise measured coincidental timing distributions, giving rise to the central resolution defining FWHM value. Compared to the earlier, long-established models [4,5], the one developed herein extends beyond the sole scintillation process, taking into account a subsequent light propagation, while proposing the simple manner in which to include even the effects of the photon detecting units. An assumption of a fully general form for the emission of the scintillation photons, their spatial distribution and subsequent attenuation inside the fiber material allow physical considerations of varying complexity to be adopted—from a simple meridional approximation to a considerably more refined description of optical processes involved. Due to the level of generality achieved, though developed with scintillating fibers in mind, the model is indiscriminately applicable to any kind of light guides, making it an acute mathematical tool for predicting the experimental results far beyond the assumptive limitations of a specific setup. As a starting point a model for the photon propagation times is considered—initially developed in Ref. [2] for bulk scintillators and later applied to scintillating fibers in Ref. [6]—from which the technical formalism was adopted.

2. Basic model

In common nuclear or particle physics experiment the arrival time of a signal is determined by the leading edge or the constant fraction discrimination of the signal's leading edge. Since the first photon impinging on the detector initiates the rise of the signal, its statistics is of the utmost importance for the description of signal timing properties.

Therefore, let us assume that inside the scintillating fiber of a length L at a distance l from one of its ends a scintillation pulse was induced (Fig. 1), emitting a total of n photons. To determine a probability f_n for the first arriving photon to reach the fiber end, several separate cases must be considered. For example, first arriving photon may correspond to the first emitted (probability p_1). On the other hand, first $m-1$ emitted ones may be lost either by escaping from the fiber before reaching its end or by absorption

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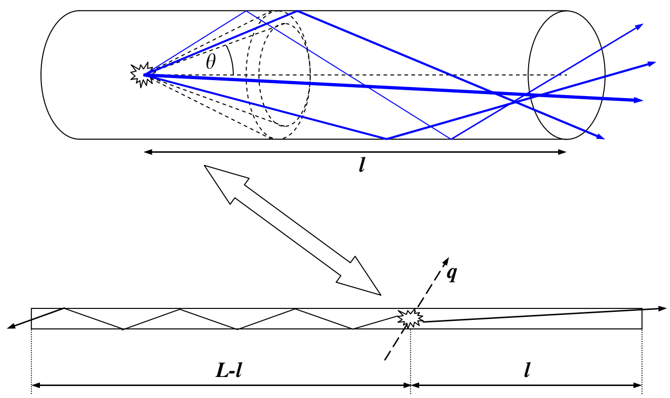

Fig. 1. A scintillating pulse induced by the charged particle q passing through the fiber at a distance l from one of its ends. The difference of initial photons' times of arrival at each end constitutes a basis for measuring the coincidental timing resolution. A wide spatial dispersion of isotropically emitted photons is also illustrated, increasing the emission probability for greater θ angles, while degrading attainable resolution.

inside the fiber material. Therefore, m-th emitted one becomes first to reach the fiber end (probability p_m) and induce the signal in the detector. Consequently, all such contributions give rise to probability f_n:

$$f_n = \sum_{m=1}^{n} p_m \tag{2.1}$$

Let us beforehand define the moment t_A for the first photon arrival at the fiber end, the moment t_E of its emission and the time t_P needed for its propagation through the fiber. The following holds:

$$t_A = t_E + t_P \tag{2.2}$$

Most simplistic in form, previous relation will be fundamental for final calculations. For a detailed analysis, let us separate contributions to p_m:

$$p_m = \alpha_m \beta_m \gamma_m \tag{2.3}$$

with α_m regulating $m-1$ photons emitted before the m-th one, β_m regulating m-th emitted one as first arriving to the fiber end, and γ_m regulating all the $n-m$ following ones. For a description of these probability contributions let us assume a completely general form $E(t_E)$ for the emission probability distribution and $S(t_P)$ for the spatial distribution of emitted photons expressed in terms of the photon time propagation, i.e. translated into the path length dispersion. Furthermore, an attenuation factor $A(t_P)$ governing the probability of single photon reaching the fiber end will be required. Defining two additional terms will prove to be most convenient for further calculations. Therefore, for the fiber excitation occurring at the moment t_E=0, let us define $I(t_E)$:

$$I(t_E) \equiv \int_0^{t_E} E(t_\varepsilon)\, \mathrm{d}t_\varepsilon \tag{2.4}$$

as a probability for photon emission prior to the moment t_E. The second useful term Λ:

$$\Lambda \equiv \int_{t_{min}}^{t_{max}} S(t_\pi) A(t_\pi)\, \mathrm{d}t_\pi \tag{2.5}$$

denotes the probability for emitted photon to actually reach the fiber end. With t_{min} as minimal time required for the photon propagation[1] and t_{max} as maximal time of propagation permitted,[2] Λ is governed by the probability for photon to become trapped by internal reflections inside the fiber and not to be absorbed within the fiber material.

For the m-th emitted photon to become the first to arrive at the fiber end, those $m-1$ previously emitted have to be lost, either by escaping the fiber or by absorption. Therefore, considering the photon combinations, factor α_m from Eq. (2.3):

$$\alpha_m = \binom{n}{m-1} [I(t_E)(1-\Lambda)]^{m-1} \tag{2.6}$$

is given by the probability for initial $m-1$ photons to be emitted prior to the emitting moment t_E of the m-th one, and subsequently not to reach the fiber end. It is to be noted that α_m was constructed without arranging the lost photons in time, which is an approach validated in Appendix A. Furthermore, after being emitted at t_E, factor β_m:

$$\beta_m = (n-m+1) E(t_E) A(t_P) S(t_P) \tag{2.7}$$

regulates the spatial direction, i.e. path length, and attenuation probability for the m-th photon. The number of remaining single photon selections is also taken into account. Finally, factor γ_m:

$$\gamma_m = [1-I(t_E)]^{n-m} \tag{2.8}$$

is restrained only by the requirement for the emission of remaining $n-m$ photons occurring after t_E, regardless of their outcome. Isolated combinatory factor is absent because all the selection options were depleted by α_m and β_m. It is to be noted that α_m and γ_m are true probabilities, while β_m is, in fact, a probability density.

With α_m, β_m, γ_m obtained, p_m is completely determined by Eq. (2.3), and f_n, consequently, by Eq. (2.1). Therefore, writing f_n explicitly:

$$f_n = E(t_E) A(t_P) S(t_P) \times \sum_{m=1}^{n} (n-m+1) \binom{n}{m-1} [I(t_E)(1-\Lambda)]^{m-1} [1-I(t_E)]^{n-m} \tag{2.9}$$

it may be noted that by shifting a summation index a step backwards, a binomial expansion remains, reducing Eq. (2.9) into:

$$f_n(t_P, t_E; l) = nE(t_E) A(t_P) S_l(t_P) [1-I(t_E)\Lambda_l]^{n-1} \tag{2.10}$$

In Eq. (2.10) an explicit dependency on the position l of a scintillation pulse origin along the fiber, the emission moment t_E and the photon propagation time t_P was written down for the purposes of further calculations.

To complete the model, a number of emitted photons per scintillation pulse must be considered. For this a simple but effective and experimentally validated Poisson statistics is employed, defining the probability $P_N(n)$ for the emission of n photons:

$$P_N(n) = \frac{N^n e^{-N}}{n!} \tag{2.11}$$

parameterized only by their mean number N per scintillation pulse. With this final distribution included, a probability density $\rho_l(t_P, t_E)$ for first arriving photon being assigned t_E and t_P may be obtained[3]:

$$\rho_l(t_P, t_E) = \sum_{n=1}^{\infty} P_N(n) f_n(t_P, t_E; l) \tag{2.12}$$

[1] Minimal propagation time t_{min} corresponds to the shortest distance path between the point of scintillation inside the fiber and the fiber end, i.e. to the photon emitted along the fiber axis.

[2] Maximal propagation time t_{max} is commonly considered to be defined by the optical condition for a total reflection off the fiber walls, i.e. by the critical angle for total reflection. However, this is not strictly true, which will be discussed in Appendix C.

[3] The formal grounds for this step are addressed in Appendix B.