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Gain of double-slab Cherenkov free-electron laser

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A B S T R A C T

A formula is derived for the small-signal gain of a double-slab Cherenkov free-electron laser. The simplified model is composed of a rectangular wave-guide partially filled with two lined parallel dielectric slabs and a sheet electron beam. The theory describes the electron beam as a plasma dielectric moving between the two dielectric slabs. With the help of hydrodynamic approximation, we derived the dispersion equation and the formula of small-signal gain. Through numerical computing, we studied an ongoing experiment of double-slab Cherenkov free-electron laser, and worked out the synchronous frequency and single-pass gain.

1. Introduction

The interest on the terahertz radiation sources keeps growing in recent years because this frequency provides wide applications in medical, industrial and material science [1–4]. Currently, there are several ways to produce terahertz radiation. Gas lasers are commercially available and can provide hundreds of lines between 40 and 1000 μm, with power around 1 W, but they are inherently not tunable. Solid-state terahertz sources usually are driven by sub-picosecond optical laser pulses. For example, a p-type Ge laser can be continuously tuned from 1 to 4 THz, but they require a large external magnetic field, and have a limited repetition rate because of crystal heating [5]. The electron beam-driven radiation sources, such as free-electron lasers, gyrotrons and synchrotrons, generate powerful terahertz wave with an average power of kW [6–8], but they need large facilities. A typical terahertz free-electron laser facility developed at KAERI [9] can provide the radiation wavelength from 100 to 300 μm, and it requires magnetic undulator and a microtron accelerator to serve an ∼6 MeV electron beam. The Cherenkov free-electron lasers have an advantage over the usual undulator free-electron lasers, and they can generate terahertz radiation with low-energy electron beam [10].

We plan to construct a compact terahertz Cherenkov free-electron laser with moderate (∼10 mW) average power. To achieve this goal, a compact electron beam source is being developed, and a double-slab Cherenkov free-electron laser resonator is being studied. To perform a preliminary lasing experiment, this resonator is designed to generate the millimeter wave. The device is composed of a rectangular wave-guide loaded with double dielectric slabs, and between them is the vacuum space for electron beam to go through. The slab is with a thickness of 650 μm and the vacuum width is 1000 μm. The overall length of the resonator is 11 cm. The dielectric medium is chosen as silicon since it has a relatively high dielectric constant, $\varepsilon_r = 11.6$, with which the device can produce radiation from millimeter to terahertz wave. The electron beam source generates a round beam with an average radius of 300 μm. The maximum beam current is 1 mA, and the energy ranges from 30 up to 50 keV.

In this paper, we aim to analyse the dispersion relation and the small-signal gain for the double-slab Cherenkov device. Based on the hydrodynamic model, the dispersion equation is derived and solved numerically, as well as the single-pass gain is worked out for the parameters of our preliminary experiment.

2. Theory

The schematic drawing of the double-slab Cherenkov free-electron laser is shown in Fig. 1. It is a two-dimensional model. A sheet electron beam with limited thickness travels in the vacuum area between the dielectric slabs; outside the dielectric slabs is the pad of perfect conductor. Only the TM mode is considered to propagate along the wave-guide and it is described by the z component of the magnetic vector potential; we study the

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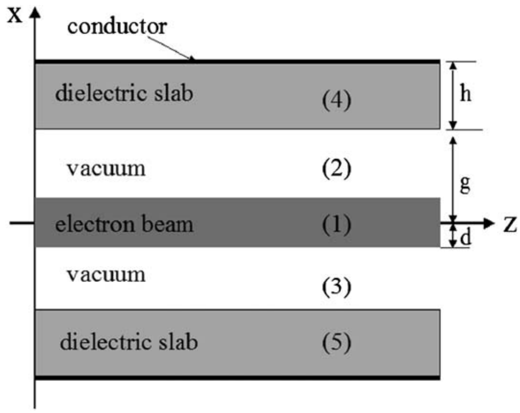

Fig. 1. The schematic of double-slab Cherenkov free-electron laser.

single-pass gain, so that the reflection of the feedback circuit is not taken into account. It is assumed that all fields are independent of the *y*-coordinate and the electrons are confined to move in the *z*-direction. We further assume that the vector potential be with the form of $A_z(x,z,t) = A_z(x)e^{-jkz}e^{i\omega t}$. From the hydrodynamic Maxwell equations, it is straightforward to obtain the wave equation including the electron beam effect, and it reads

$$\left(\frac{d^2}{dx^2} + \left(\frac{\omega^2}{c^2} - k^2\right)\left(1 - \frac{\omega_p^2}{\gamma^3(\omega - kv_0)^2}\right)\right) A_z(x) = 0, \tag{1}$$

where ω_p is the plasma frequency, v_0 the electron beam velocity, γ the relativistic factor and c the light velocity in vacuum. The expressions of the electric field and magnetic field can be derived through $\vec{B} = \nabla \times \vec{A}$ and $\vec{E} = -j\omega \vec{A} - \nabla \times \varphi$, where φ is the scalar potential that can be achieved by Lorentz gauge. The interaction area is divided into five regions as noted in Fig. 1. By solving Eq. (1), we derived the electromagnetic fields in the electron beam region (1) and they are

$$E_{z,1} = \frac{c^2\Gamma_1^2}{j\omega}(C_{1,1}\sin(\Gamma_1 x) + C_{1,2}\cos(\Gamma_1 x)) \tag{2}$$

$$H_{y,1} = \frac{\Gamma_1}{\mu_0}(-C_{1,1}\cos(\Gamma_1 x) + C_{1,2}\sin(\Gamma_1 x)) \tag{3}$$

where

$$\Gamma_1^2 = \left(\frac{\omega^2}{c^2} - k^2\right)\left(1 - \frac{\omega_p^2}{\gamma^3(\omega - kv_0)^2}\right).$$

In the vacuum regions (2) and (3), the electron beam vanishes and Eq. (1) becomes simple by $\omega_b = 0$, then the electromagnetic fields read

$$E_{z,2} = \frac{c^2\Gamma_2^2}{j\omega}(C_{2,1}\sin(\Gamma_2 x) + C_{2,2}\cos(\Gamma_2 x)) \tag{4}$$

$$H_{y,2} = \frac{\Gamma_2}{\mu_0}(-C_{2,1}\cos(\Gamma_2 x) + C_{2,2}\sin(\Gamma_2 x)) \tag{5}$$

and

$$E_{z,3} = \frac{c^2\Gamma_3^2}{j\omega}(C_{3,1}\sin(\Gamma_3 x) + C_{3,2}\cos(\Gamma_3 x)) \tag{6}$$

$$H_{y,3} = \frac{\Gamma_3}{\mu_0}(-C_{3,1}\cos(\Gamma_3 x) + C_{3,2}\sin(\Gamma_3 x)) \tag{7}$$

where $\Gamma_2^2 = \Gamma_3^2 = \omega^2/c^2 - k^2$.

In a similar way in the dielectric regions (4) and (5), the electromagnetic fields are expressed as

$$E_{z,4} = \frac{c^2\Gamma_4^2}{j\omega\varepsilon_r}(C_{4,1}\sin(\Gamma_4 x) + C_{4,2}\cos(\Gamma_4 x)) \tag{8}$$

$$H_{y,4} = \frac{\Gamma_4}{\mu_0}(-C_{4,1}\cos(\Gamma_4 x) + C_{4,2}\sin(\Gamma_4 x)) \tag{9}$$

and

$$E_{z,5} = \frac{c^2\Gamma_5^2}{j\omega\varepsilon_r}(C_{5,1}\sin(\Gamma_5 x) + C_{5,2}\cos(\Gamma_5 x)) \tag{10}$$

$$H_{y,5} = \frac{\Gamma_5}{\mu_0}(-C_{5,1}\cos(\Gamma_5 x) + C_{5,2}\sin(\Gamma_5 x)) \tag{11}$$

where $\Gamma_4^2 = \Gamma_5^2 = \varepsilon_r\omega^2/c^2 - k^2$.

Considering the boundary conditions, at $x = g+h$ and $x = -(g+h)$, the tangential electric field should be zero, i.e., $E_{z,4} = E_{z,5} = 0$, then we get

$$C_{4,2} = -C_{4,1}\tan(\Gamma_4(g+h)) \tag{12}$$

$$C_{5,2} = C_{5,1}\tan(\Gamma_5(g+h)) \tag{13}$$

Due to the continuity of electromagnetic field, we have $E_{z,2} = E_{z,4}$, $H_{y,2} = H_{y,4}$ at $x = g$. And considering Eq. (12), we get

$$\Gamma_2^2(C_{2,1}\sin(\Gamma_2 g) + C_{22}\cos(\Gamma_2 g)) = C_{4,1}\frac{\Gamma_4^2}{\varepsilon_r}(\sin(\Gamma_4 g) - \tan(\Gamma_4(g+h))\cos(\Gamma_4 g)) \tag{14}$$

$$\Gamma_2(-C_{2,1}\cos(\Gamma_2 g) + C_{22}\sin(\Gamma_2 g)) = C_{4,1}\Gamma_4(-\cos(\Gamma_4 g) - \tan(\Gamma_4(g+h))\sin(\Gamma_4 g)) \tag{15}$$

Let us combine Eqs. (14) and (15) into a single equality, and it reads

$$C_{2,1}(\sin(\Gamma_2 g) + M\cos(\Gamma_2 g)) + C_{2,2}(\cos(\Gamma_2 g) - M\sin(\Gamma_2 g)) = 0 \tag{16}$$

where

$$M = \frac{\Gamma_4}{\Gamma_2\varepsilon_r}\frac{\sin(\Gamma_4 g) - \tan(\Gamma_4(g+h))\cos(\Gamma_4 g)}{-\cos(\Gamma_4 g) - \tan(\Gamma_4(g+h))\sin(\Gamma_4 g)}$$

In a similar way, for the boundary of $x = -g$, by using $E_{z,3} = E_{z,5}$, $H_{y,3} = H_{y,5}$ and expression (13), we get

$$C_{3,1}(-\sin(\Gamma_3 g) - N\cos(\Gamma_3 g)) + C_{3,2}(\cos(\Gamma_3 g) - N\sin(\Gamma_3 g)) = 0 \tag{17}$$

where

$$N = \frac{\Gamma_5}{\Gamma_3\varepsilon_r}\frac{\sin(\Gamma_5 g) - \tan(\Gamma_5(g+h))\cos(\Gamma_5 g)}{-\cos(\Gamma_5 g) - \tan(\Gamma_5(g+h))\sin(\Gamma_5 g)}.$$

Based on the continuity of electromagnetic field at $x = \pm d$, i.e., $E_{z,1} = E_{z,2}$, $H_{y,1} = H_{y,2}$, $E_{z,1} = E_{z,3}$ and $H_{y,1} = H_{y,3}$, the following expressions are achieved:

$$C_{1,1}\Gamma_1^2\sin(\Gamma_1 d) + C_{1,2}\Gamma_1^2\cos(\Gamma_1 d) + C_{2,1}(-\Gamma_2^2\sin(\Gamma_2 d)) + C_{2,2}(-\Gamma_2^2\cos(\Gamma_2 d)) = 0 \tag{18}$$

$$C_{1,1}(-\Gamma_1\cos(\Gamma_1 d)) + C_{1,2}\Gamma_1\sin(\Gamma_1 d) + C_{2,1}\Gamma_2\cos(\Gamma_2 d) + C_{2,2}(-\Gamma_2\sin(\Gamma_2 d)) = 0 \tag{19}$$

$$C_{1,1}\Gamma_1^2\sin(\Gamma_1 d) + C_{1,2}\Gamma_1^2\cos(\Gamma_1 d) + C_{3,1}\Gamma_3^2\sin(\Gamma_3 d) + C_{2,2}(-\Gamma_3^2\cos(\Gamma_3 d)) = 0 \tag{20}$$