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Nuclear Physics B 866 (2013) 72–85
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

The gravity dual of supersymmetric gauge theories on a biaxially squashed three-sphere

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Received 16 July 2012; accepted 20 August 2012

Available online 23 August 2012

Abstract

We present the gravity dual to a class of three-dimensional $\mathcal{N}=2$ supersymmetric gauge theories on a biaxially squashed three-sphere, with a non-trivial background gauge field. This is described by a $1/2$ BPS Euclidean solution of four-dimensional $\mathcal{N}=2$ gauged supergravity, consisting of a Taub–NUT–AdS metric with a non-trivial instanton for the graviphoton field. The holographic free energy of this solution agrees precisely with the large N limit of the free energy obtained from the localized partition function of a class of Chern–Simons quiver gauge theories. We also discuss a different supersymmetric solution, whose boundary is a biaxially squashed Lens space $S^3/\mathbb{Z}_2$ with a topologically non-trivial background gauge field. This metric is of Eguchi–Hanson–AdS type, although it is not Einstein, and has a single unit of gauge field flux through the S^2 cycle.

Keywords: Supersymmetry; Localization; AdS/CFT correspondence

1. Introduction

Supersymmetric gauge theories on compact curved backgrounds are interesting for various reasons. For example, supersymmetry may be combined with localization techniques, allowing one to perform a variety of exact computations in strongly coupled field theories. The authors of [1] presented a construction of $\mathcal{N}=2$ supersymmetric gauge theories in three dimensions in the background of a $(U(1)\times U(1))$-invariant squashed three-sphere and R-symmetry gauge field.

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0550-3213/$ – see front matter
http://dx.doi.org/10.1016/j.nuclphysb.2012.08.015

The gravity dual of this construction was recently given in [2]. It consists of a 1/4 BPS Euclidean solution of four-dimensional $\mathcal{N}=2$ gauged supergravity, which in turn may be uplifted to a supersymmetric solution of eleven-dimensional supergravity. In particular, the bulk metric in [2] is simply AdS_4, and the graviphoton field is an instanton with (anti)-self-dual field strength. The asymptotic metric and gauge field then reduce to the background considered in [1].

The purpose of this letter is to present the gravity dual to a different field theory construction, obtained recently in [3]. In this reference the authors have constructed three-dimensional $\mathcal{N}=2$ supersymmetric gauge theories in the background of the $(SU(2)\times U(1))$-invariant squashed three-sphere (which we refer to as *biaxially* squashed) and a non-trivial background $U(1)$ gauge field, and have computed the corresponding partition functions using localization. Differently from a similar construction discussed briefly in [1], this partition function depends non-trivially on the squashing parameter. As we will see, the gravity dual to this set-up will have some distinct features with respect to the solution in [2]. In particular, the metric is not simply AdS_4, although it will again be an Einstein metric, and there is a self-dual graviphoton.

The plan of the rest of this paper is as follows. In Section 2 we review the construction of [3]. In Section 3 we discuss the gravity dual. In Section 4 we describe a different supersymmetric solution, consisting of a non-Einstein metric and a non-instantonic graviphoton field. Section 5 concludes.

2. Supersymmetric gauge theories on the biaxially squashed S^3

In the construction of [3] the metric on the three-sphere is, up to an irrelevant overall factor, given by

$$\mathrm{d}s_3^2=\sigma_1^2+\sigma_2^2+\frac{1}{v^2}\sigma_3^2, \tag{2.1}$$

where σ_i are the standard $SU(2)$ left-invariant one-forms on S^3, defined as $\mathrm{i}\sigma_i\tau_i=-2\mathrm{g}^{-1}\,\mathrm{d}\mathrm{g}$, where τ_i denote the Pauli matrices and $\mathrm{g}\in SU(2)$. The background $U(1)$ gauge field reads

$$A^{(3)}=\frac{\sqrt{v^2-1}}{2v^2}\sigma_3, \tag{2.2}$$

and the spinors in the supersymmetry transformations obey the equation (setting the radius $r=2$ in [3])

$$\nabla_\alpha^{(3)}\chi-\frac{\mathrm{i}}{4v}\gamma_\alpha\chi-A_\beta^{(3)}\gamma_\alpha{}^\beta\chi=0, \tag{2.3}$$

where $\nabla_\alpha^{(3)}$, $\alpha=1,2,3$, is the spinor covariant derivative constructed from the metric (2.1), and γ_α generate $\mathrm{Cliff}(3,0)$. There are *two* linearly independent solutions to (2.3), transforming as a doublet under $SU(2)$, whose explicit form is given in [3]. This will be important for identifying the gravity dual.

In [3] the authors constructed Chern–Simons, Yang–Mills, and matter Lagrangians for the $\mathcal{N}=2$ vector multiplets $V=(\mathscr{A}_\alpha,\sigma,\lambda,D)$ and chiral multiplets $\Phi=(\phi,\psi,F)$, in the background of the metric (2.1) and R-symmetry gauge field (2.2). These are invariant under a set of supersymmetry transformations, provided the spinorial parameters obey Eq. (2.3). The supersymmetric completion of the Chern–Simons Lagrangian contains new terms, in addition to those appearing in flat space, proportional to σ^2 and $\sigma A^{(3)}\wedge\mathrm{d}\mathscr{A}$ (cf. Eq. (32) of [3]). The Yang–Mills and matter Lagrangians are total supersymmetry variations (cf. Eq. (31) of [3]) and therefore can