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Sigma models in the presence of dynamical point-like defects

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Abstract

Point-like Liouville integrable dynamical defects are introduced in the context of the Landau–Lifshitz and Principal Chiral (Faddeev–Reshetikhin) models. Based primarily on the underlying quadratic algebra we identify the first local integrals of motion, the associated Lax pairs as well as the relevant sewing conditions around the defect point. The involution of the integrals of motion is shown taking into account the sewing conditions.

MSC: Classical integrability; Integrable defects; Sigma models

1. Introduction

The issue of integrable defects has been quite intriguing, and a considerable number of studies have been devoted to this particular problem, both at classical and quantum level [1–22]. From a physical point of view, the insertion of defects or impurities within a given physical system renders the latter more interesting and realistic. Applications of such studies are important for a number of different disciplines, including models in condensed matter theory (see e.g. [1,23,24]) and quantum information (see e.g. [25,26]). It is desirable then to introduce defects in integrable theories, in such a way that integrability is preserved, so that the corresponding tools can be used in order to obtain precise information about the defect model.

A systematic algebraic formulation for describing Liouville integrable point-like defects was recently introduced in [21,22]. The description was primarily based on the underlying quadratic

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algebra satisfied by the bulk monodromy matrices, which describe the left and right theories, as well as by the defect L-matrix. In fact, this is the main necessary requirement so that Liouville integrability may be by construction guaranteed. The main steps of the proposed algebraic methodology are provided in the subsequent sections, however for a more detailed description of the process we refer the interested reader to [21,22].

An efficient description of defects becomes more intricate at the level of classical integrable field theories, where usually the defect is introduced as a discontinuity (*jump*) together with suitable sewing conditions [8,10]. In the present scheme the sewing conditions naturally emerge as continuity conditions on the time components of the Lax pair around the defect point. It was also rigorously proven in [21] that the sewing conditions are compatible with the hierarchy of the Hamiltonians, a fact that ensures that the proposed formulation is well defined and consistent.

In the present work, we consider dynamical point-like defects in the context of sigma models, such as the Landau–Lifshitz (LL) model and a variation of the familiar principal chiral model (PCM), the so-called Faddeev–Reshetikhin (FR) [27] model, in such a way that integrability is preserved. It is worth noting that this kind of systems, in addition to their own physical and mathematical value have also attracted considerable interest lately due to the fact that they typically arise within the AdS/CFT context (see e.g. [28–30] and references therein). Thus, it would be of great significance to investigate possible relevant physical implications in this particular frame. Recall also that the typical PCM possesses a non-ultra-local algebra rendering its quantization a very intricate task, whereas its variation, the FR model is associated to a more familiar ultra-local algebra, much easier to deal with. In any case, both PCM and FR models share the same Lax pair, and consequently the same equations of motion, hence they are physically quite similar. Therefore, the findings presented for the FR model are naturally relevant for the conventional PCM model.

Based on the formulation of [21,22] we introduce the defect matrix, and the associated modified monodromy matrix. Using this as our starting point we extract the first couple of the local integrals of motion, and the corresponding time components of the Lax pairs for both models. Due to analyticity requirements imposed on the time components of the Lax pairs, certain sewing conditions emerge at the defect point. The involution of the charges is shown based on the underlying algebra, and the invariance of the sewing conditions under the Hamiltonian action is also explicitly checked.

2. The Landau–Lifshitz model with defect

We first consider integrable point-like dynamical defects in the context of the isotropic Landau–Lifshitz model. We assume periodic boundary conditions and restrict our attention to the $\mathfrak{su}_2$ classical algebra [31]. Results regarding higher rank algebras may also be considered [32].

The equations of motion associated with the isotropic Landau–Lifshitz model are given as:

$$\frac{\partial \vec{S}}{\partial t} = i\vec{S} \times \frac{\partial^2 \vec{S}}{\partial x^2}, \tag{2.1}$$

where the vector-valued functions $\vec{S}(x) = (S_1(x), S_2(x), S_3(x))$ which describe the physical quantities of the model take values on the unit 2-sphere, i.e. $\vec{S} \cdot \vec{S} = 1$, and satisfy an $\mathfrak{su}_2$ Poisson structure given by the following Poisson brackets

$$\{S_a(x),\ S_b(y)\} = 2i\varepsilon_{abc} S_c(x)\delta(x-y), \tag{2.2}$$