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Nuclear Physics B 707 [FS] (2005) 493–508

NUCLEAR PHYSICS B

Large-order asymptotes for dynamic models near equilibrium

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Received 21 June 2004; accepted 10 November 2004

Available online 26 November 2004

Abstract

Instanton analysis is applied to model A of critical dynamics. It is shown that the static instanton of the massless ϕ^4 model determines the large-order asymptotes of the perturbation expansion of the dynamic model.

PACS: 11.10.Jj; 05.70.Jk

Keywords: Instanton; Dynamic models; Large orders

1. Introduction

The knowledge of large-order asymptotic behaviour of perturbation series of static field-theoretic models is important for resummation of series for critical exponents and scaling functions [1]. This behaviour has been thoroughly explored with the aid of instanton analysis and applied to the resummation problem in the prototypical static ϕ^4 model [2–4],

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2004.11.016

which has been widely used as a model of critical behaviour in continuous phase transitions of ferromagnetic type.

However, little is known about large-order asymptotics in dynamic field theories constructed from Langevin equations with the aid of the Martin–Siggia–Rose (MSR) formalism [5]. Recently, it was stated [6,7] that there is no instanton within the MSR approach in the Kraichnan model—which has attracted considerable attention as a model describing intermittency in turbulent diffusion [8]—and that the method of steepest descent has to be used in Lagrangean variables.

In this paper we propose a method to assess large-order asymptotic behaviour of dynamic models near equilibrium, i.e., with Gibbsian static limit (we also will restrict ourselves to models generated by Langevin equations without mode coupling). We will use the steepest descent method (instanton approach) to find the large-order behaviour in a representative model. We will discuss large-order asymptotes of correlation functions, response functions and critical exponents. For equal-time correlation functions this asymptotic behaviour is shown to coincide with that of the static instanton approach.

The particular model we deal with in this paper is one of the standard dynamic ϕ^4-based models: model A in the classification of Ref. [9]. In this model critical exponents are the same as in the static ϕ^4 model apart from the dynamic exponent z. The large-order asymptotics of the dynamic exponent have not been analyzed so far. It should be noted here that the use of Lagrangean variables becomes prohibitively difficult in this case due to the essential nonlinearity of the problem (the Kraichnan model is linear in the advected scalar field).

The present article is organized as follows: in Section 2 construction of the MSR field theory corresponding to a nonlinear Langevin equation is reviewed with special emphasis on the treatment of the functional determinant for the steepest descent method. Existence of the dynamic instanton and relation of the dynamic instanton solution to the static one is analysed in Section 3. The fluctuation determinant is calculated in Section 4, whereas Section 5 is devoted to a brief analysis of correlation and response functions. Results of this paper are summarized in Section 6.

2. Dynamic field theory

Consider the Langevin equation

$$\frac{\partial\varphi}{\partial t}+\Gamma\frac{\delta S}{\delta\varphi}=\xi, \tag{1}$$

where ξ is a Gaussian random field with zero mean and the correlation function

$$\left\langle\xi(t,\mathbf{x})\xi(t',\mathbf{x}')\right\rangle=D(x-x')=2\Gamma\delta(t-t')\delta(\mathbf{x}-\mathbf{x}'),$$

where the shorthand notation $x=(t,\mathbf{x})$ has been used. In Eq. (1) the action is the static action of arbitrary model with the known instanton. The most interesting example is the massless φ^4 model:

$$S=\frac{1}{2}\partial\varphi\partial\varphi+\frac{g}{4!}\varphi^4. \tag{2}$$