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A_{n-1} Gaudin model with open boundaries

Wen-Li Yang [a,b], Yao-Zhong Zhang [b], Ryu Sasaki [c]

[a] *Institute of Modern Physics, Northwest University, Xian 710069, PR China*
[b] *Department of Mathematics, University of Queensland, Brisbane, QLD 4072, Australia*
[c] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

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Abstract

The A_{n-1} Gaudin model with integrable boundaries specified by *non-diagonal* K-matrices is studied. The commuting families of Gaudin operators are diagonalized by the algebraic Bethe ansatz method. The eigenvalues and the corresponding Bethe ansatz equations are obtained.

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1. Introduction

Gaudin type models constitute a particularly important class of one-dimensional many-body systems with long-range interactions. They have found applications in many branches of fields ranging from condensed matter physics to high energy physics. For example, Gaudin models have been used to establish the integrability of the reduced BCS theory of small metallic grains [1–4] and the Seiberg–Witten supersymmetric Yang–Mills theory [5]. They have also provided a powerful tool for constructing the solutions to the Knizhnik–Zamolodchikov equation [6–10] of the Wess–Zumino–Novikov–Witten conformal field theory.

Recently Gaudin models with non-trivial boundaries have attracted much interest [9,11–15]. So far, attention has largely been concentrated on Gaudin models with boundary conditions specified by *diagonal* K-matrices. In [12], the XXZ Gaudin model with boundaries given by the *non-diagonal* K-matrices in [16,17] was constructed and solved by the algebraic Bethe ansatz

E-mail address: yzz@maths.uq.edu.au (Y.-Z. Zhang).

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method. In this paper we generalize the results in [12] and solve the A_{n-1} Gaudin magnets with open boundary conditions corresponding to the *non-diagonal* K-matrices obtained in [18].

This paper is organized as follows. In Section 2, we briefly review the inhomogeneous $A^{(1)}_{n-1}$ trigonometric vertex model with integrable boundaries, which also services as introducing our notation and some basic ingredients. In Section 3, we construct the generalized Gaudin operators associated with non-diagonal K-matrices. The commutativity of these operators follows from applying the standard procedure [9,11,19,20] to the inhomogeneous $A^{(1)}_{n-1}$ trigonometric vertex model with off-diagonal boundaries found in [18], thus ensuring the integrability of the Gaudin magnets. In Section 4, we diagonalize the Gaudin operators simultaneously by means of the algebraic Bethe ansatz method. This constitutes the main new result of this paper. The diagonalization is achieved by means of the technique of the "vertex-face" transformation [21]. Section 5 is for conclusions. In Appendix A, we list the explicit matrix expressions of the K-matrices corresponding to the $n = 3, 4$ cases.

2. Preliminaries: inhomogeneous $A^{(1)}_{n-1}$ open chain

Let us fix a positive integer n ($n \geqslant 2$) and a generic complex number η, and $R(u) \in \mathrm{End}(\mathbb{C}^n \otimes \mathbb{C}^n)$ be the R-matrix of the $A^{(1)}_{n-1}$ trigonometric vertex model given by [22–24]

$$R(u) = \sum_{\alpha=1}^{n} R^{\alpha\alpha}_{\alpha\alpha}(u) E_{\alpha\alpha} \otimes E_{\alpha\alpha} + \sum_{\alpha\neq\beta} \left\{ R^{\alpha\beta}_{\alpha\beta}(u) E_{\alpha\alpha} \otimes E_{\beta\beta} + R^{\beta\alpha}_{\alpha\beta}(u) E_{\beta\alpha} \otimes E_{\alpha\beta} \right\}, \tag{2.1}$$

where E_{ij} is the matrix with elements $(E_{ij})^l_k = \delta_{jk}\delta_{il}$. The coefficient functions are

$$R^{\alpha\beta}_{\alpha\beta}(u) = \begin{cases} \frac{\sin(u)}{\sin(u+\eta)} e^{-i\eta}, & \alpha > \beta, \\ 1, & \alpha = \beta, \\ \frac{\sin(u)}{\sin(u+\eta)} e^{i\eta}, & \alpha < \beta, \end{cases} \tag{2.2}$$

$$R^{\beta\alpha}_{\alpha\beta}(u) = \begin{cases} \frac{\sin(\eta)}{\sin(u+\eta)} e^{iu}, & \alpha > \beta, \\ 1, & \alpha = \beta, \\ \frac{\sin(\eta)}{\sin(u+\eta)} e^{-iu}, & \alpha < \beta. \end{cases} \tag{2.3}$$

The R-matrix satisfies the quantum Yang–Baxter equation (QYBE)

$$R_{12}(u_1 - u_2) R_{13}(u_1 - u_3) R_{23}(u_2 - u_3) = R_{23}(u_2 - u_3) R_{13}(u_1 - u_3) R_{12}(u_1 - u_2), \tag{2.4}$$

and the properties [18]:

$$\text{Unitarity:} \quad R_{12}(u) R_{21}(-u) = \mathrm{id}, \tag{2.5}$$

$$\text{Crossing-unitarity:} \quad R^{t_2}_{12}(u) M_2^{-1} R^{t_2}_{21}(-u - n\eta) M_2 = \frac{\sin(u)\sin(u + n\eta)}{\sin(u+\eta)\sin(u + n\eta - \eta)} \,\mathrm{id}, \tag{2.6}$$

$$\text{Quasi-classical property:} \quad R_{12}(u)\big|_{\eta\to 0} = \mathrm{id}. \tag{2.7}$$

Here $R_{21}(u) = P_{12} R_{12}(u) P_{12}$ with P_{12} being the usual permutation operator and t_i denotes the transposition in the ith space, and η is the so-called crossing parameter. The crossing matrix M