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Electromagnetic interactions of higher-dimensional quantum Hall droplets

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Abstract

Using a W_N-gauge theory to describe electromagnetic interactions of spinless fermions in the lowest Landau level, where the W_N transformations are nonlinear realizations of $U(1)$ gauge transformations, we construct the effective action describing electromagnetic interactions of a higher-dimensional quantum Hall droplet. We also discuss how this is related to the Abelian Seiberg–Witten map. Explicit calculations are presented for the quantum Hall effect on $\mathbf{CP}^k$ with $U(1)$ background magnetic field. The bulk action is a Kähler–Chern–Simons term whose anomaly is canceled by a boundary contribution so that gauge invariance is explicitly satisfied.

1. Introduction

The study of quantum Hall effect in higher dimensions has recently attracted a lot of attention [1–9] following the original work of Zhang and Hu [1]. Similarly to the classic two-dimensional QHE it provides a framework for new ideas about bosonization, topological field theories, noncommutative geometry and D-branes, in addition to the possibility of higher spin theories and its connection to the observed spin-Hall effect [10]. Zhang and Hu considered the Landau problem for charged fermions on S^4 with a background magnetic field corresponding to the standard $SU(2)$ instanton. This leads to a many-body picture in terms of incompressible quantum Hall droplets, which give rise to gapless edge excitations, in a way similar to the two-dimensional QHE.

Recently we generalized the Zhang, Hu construction to arbitrary even dimensions by formulating the quantum Hall effect on the even-dimensional, complex projective spaces $\mathbf{CP}^k$ [2].

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Within this framework, we also presented a method for obtaining the effective action for edge excitations of a quantum Hall droplet in any dimension [3,4], based on a collective formulation of the quantum mechanical density matrix in the limit of large number of fermions or equivalently a large droplet [11].

In this paper we will analyze the electromagnetic interactions (in addition to the uniform magnetic field necessary for the formation of the QHE) of the underlying charged fermions, by coupling the higher-dimensional quantum Hall system, restricted to the lowest Landau level, to a weak electromagnetic field. We are particularly interested in deriving the bulk effective action in terms of the external electromagnetic field and its relation to the edge boundary action.

It is well known that in the case of two dimensions, one can directly integrate out the fermions and derive the bulk effective action which is essentially an Abelian Chern–Simons action with its coefficient given by the quantized Hall conductance. Further this action and the boundary action are related by the property of anomaly cancellation. The Chern–Simons action defined on a space with boundary is not gauge invariant, the non-invariance given in terms of a surface term. The requirement of gauge invariance implies the presence of a boundary action with the appropriate gauge transformation. It was first shown by Wen based on the anomaly cancellation property that the edge dynamics of a two-dimensional Hall droplet is described in terms of massless chiral fields [12,13].

Our approach here is somewhat different. Starting from a very generic matrix formulation of the lowest Landau level Hall droplet dynamics, we explicitly derive the bulk effective action involving the perturbative electromagnetic fields. The electromagnetic interaction to the edge degrees of freedom is also obtained. One can show that the total action, bulk and edge, is explicitly gauge invariant. The strategy we follow is an adaptation of a method used by Sakita [14,15] to derive the electromagnetic interactions of LLL electrons in the two-dimensional plane based on the W_∞-gauge field theory [13,16].

This paper is organized as follows. In Section 2 we present a matrix formulation of the dynamics of the lowest Landau level with and without electromagnetic interactions and we outline a general method for deriving the corresponding effective action of a quantum Hall droplet in arbitrary number of dimensions. In the rest of the paper we present an explicit analysis for the case of a quantum Hall droplet on $\mathbf{CP}^k$ with a $U(1)$ background field. In Section 3 we briefly review the structure of the lowest Landau level and the emerging star product for $\mathbf{CP}^k$ with $U(1)$ background magnetic field. In Section 4 we present the derivation of the effective action in the absence of electromagnetic interactions and in Section 5 we extend this to include electromagnetic interactions. The role of the Seiberg–Witten map in the derivation of the effective action is discussed. In Section 6 we analyze the gauge invariance of the effective action and explicitly show how the anomaly of the bulk part, which turns out to be a Kähler–Chern–Simons term, is canceled by the boundary contribution. A brief conclusion and comments are presented in Section 7.

2. General approach

We shall first review the method used in [3,4] to derive the action describing the dynamics of a generic lowest Landau level Hall droplet in the absence of external weak electromagnetic interactions and then discuss how this gets modified in order to describe electromagnetic interactions and derive the corresponding effective action.

Let N denote the dimension of the one-particle Hilbert space corresponding to the states of the lowest Landau level, K of which are occupied by fermions. Spin degrees of freedom are neglected, so each state can be occupied by a single fermion. In the presence of a confining