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The self-gravitating gas in the presence of dark energy: Monte Carlo simulations and stability analysis

H.J. de Vega [a,b], J.A. Siebert [a]

[a] *Laboratoire de Physique Théorique et Hautes Energies, Université Paris VI et Paris VII, Laboratoire Associé au CNRS UMR 7589, Tour 24, 5ème étage, 4 Place Jussieu, 75252 Paris cedex 05, France*
[b] *Observatoire de Paris, LERMA, Laboratoire Associé au CNRS UMR 8112, 61 Avenue de l'Observatoire, 75014 Paris, France*

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Abstract

The self-gravitating gas in the presence of a positive cosmological constant Λ is studied in thermal equilibrium by Monte Carlo simulations and by the mean field approach. We find excellent agreement between both approaches already for $N = 1000$ particles on a volume V (the mean field is exact in the infinite N limit). The domain of stability of the gas is found to increase when the cosmological constant increases. The particle density is shown to be an increasing (decreasing) function of the distance when the dark energy dominates over self-gravity (and vice versa). We confirm the validity of the thermodynamic limit: $N, V \to \infty$ with $N/V^{1/3}$ and $\Lambda V^{2/3}$ fixed. In such dilute limit extensive thermodynamic quantities like energy, free energy, entropy turn to be proportional to N. We find that the gas is stable till the isothermal compressibility diverges. Beyond this point the gas becomes a extremely dense object whose properties are studied by Monte Carlo.

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1. Introduction

The self-gravitating gas in thermal equilibrium has been thoroughly studied since many years [2,3,5–10,14,15]. As a consequence of the long range attractive Newton force, the self-gravitating gas admits a consistent thermodynamic limit $N, V \to \infty$ with $N/V^{1/3}$ fixed. In this

E-mail address: devega@lpthe.jussieu.fr (H.J. de Vega).

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limit, extensive thermodynamic quantities like energy, free energy, entropy are proportional to N [2,3].

In Ref. [4] we investigated how a cosmological constant affects the properties of the non-relativistic self-gravitating gas in thermal equilibrium by mean field methods. The mean field approximation becomes exact in the limit when the number of particles becomes infinity.

In the present paper we study the stability properties of the self-gravitating gas in the presence of a cosmological constant by mean field and Monte Carlo methods. The use of Monte Carlo simulations is particularly useful since they are like real experiments and allow to unambiguously determine the stability or instability of the system.

We compute both by Monte Carlo and mean field methods several physical quantities: the equation of state, the particle density and the average particle distance for different values of the dark energy. An excellent agreement between both approaches is found except very close to the collapse transition (see Figs. 4–6). The difference between both approaches turns to be, as expected, of the order $1/N$, where the number of particles N we choose in the Monte Carlo simulations was $N = 1000$–2000. The slightly larger split between Monte Carlo and mean field near the collapse is due to the fact that there the corrections to the mean field become singular [2,3].

We find that the onset of instability in the canonical ensemble coincides with the point where the isothermal compressibility diverges. At this point we find that the dimensionless parameter $\zeta = G^3 m^6 N^2 P/T^4$ is maximal (ζ was introduced in Ref. [10] for $\Lambda = 0$). We find that the domain of stability of the gas increases for increasing cosmological constant. The dark energy has an antigravity effect that disfavours the collapse pushing apart the particles.

In absence of cosmological constant the particle density $\rho(r)$ is a **decreasing** function of the distance in the self-gravitating gas (see, for example, [2]). In the presence of a positive Λ we find that $\rho(r)$ **decreases** with r when the self-gravity dominates over the dark energy. This happens for $X \equiv \frac{2\Lambda V}{mN} < 1$. For $X > 1$ the cosmological constant dominates over the self-gravity and the particle density turns to **increase** with the distance. This is a consequence of the repulsive character of the dark energy.

The Monte Carlo study of the self-gravitating gas shows the validity of the dilute thermodynamic limit introduced in Refs. [2,4]. Namely, for $N, V \to \infty$ with $N/V^{1/3}$ and $\Lambda V^{2/3}$ fixed, a consistent thermodynamic limit is reached where extensive thermodynamic quantities like energy, free energy, entropy are shown to be proportional to N. Furthermore, the Monte Carlo simulations allow us to study the condensed phase which turns to be an extremely dense body. The phase transition to collapse is found to be of zeroth order.

The outline of the paper is as follows. Section 2 presents non-relativistic self-gravitating particles in the presence of the cosmological constant, their statistical mechanical treatment and the mean field approach. In Section 3 we analyze the stability of the self-gravitating gas for zero and nonzero cosmological constant Λ while in Section 4 we give the physical picture of the gas and its particle density as a function of Λ. Section 5 contains the results of our Monte Carlo simulations on the phase diagram, the collapse transition, the particle density and the properties of the collapsed phase.