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Quantum gravity stability of isotropy in homogeneous cosmology

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ABSTRACT

It has been shown that anisotropy of homogeneous spacetime described by the general Kasner metric can be damped by quantum fluctuations coming from perturbative quantum gravity in one-loop approximation. Also, a formal argument, not limited to one-loop approximation, is put forward in favor of stability of isotropy in the exactly isotropic case.

1. Introduction

Standard [1] and loop [2–4] quantum cosmology heavily depends on the implicit assumption of (quantum) stability of general form of the metric. As a principal starting point in quantum cosmology, one usually chooses a metric of a particular (more or less symmetric) form. In the simplest, homogeneous and isotropic case, the metric chosen is the (flat) Friedmann–Lemaître–Robertson–Walker (FLRW) one. Consequently, (field theory) quantum gravity reduces to a much more tractable quantum mechanical system with a finite number of degrees of freedom. It is obvious that such an approach greatly simplifies quantum analysis of cosmological evolution, but under no circumstances is it obvious to what extent is such an approach reliable. The quantum cosmology approach could be considered unreliable when (for example) the assumed symmetry of the metric would be unstable due to quantum fluctuations. More precisely, in the context of the stability, one can put forward the two, to some extent complementary, issues (questions): (1) assuming a small anisotropy in the almost isotropic cosmological model, have quantum fluctuations a tendency to increase the anisotropy or, just the opposite, to reduce it? (2) assuming we start quantum evolution from an exactly isotropic metric should be we sure that no quantum fluctuations are able to perturb the isotropy?

In this Letter, we are going to address the both issues of the quantum stability of spacetime metric in the framework of standard covariant quantum gravity. Namely, in Section 2, we address the first stability issue for an anisotropic (homogeneous) metric of the Kasner type, to one-loop in perturbative expansion. In Section 3, we give a simple, formal argument, not limited to one-loop, concerning the second issue.

2. One-loop stability

The approach applied in this section is a generalization of our approach used in [5] in the context of FLRW geometry. In our present work, the starting point is an anisotropic (homogeneous) metric,

$$ds^2 = dt^2 - a_1^2(t)\left(dx^1\right)^2 - a_2^2(t)\left(dx^2\right)^2 - a_3^2(t)\left(dx^3\right)^2, \tag{1}$$

of the Kasner type, i.e.

$$a_i^2(t) \equiv \left|\frac{t}{t_0}\right|^{2k_i}, \quad i = 1, 2, 3, \tag{2}$$

where k_i are the Kasner exponents. One should stress that we ignore any assumptions concerning matter content, and consequently, no prior bounds are imposed on k_i.

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In the perturbative approach

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{3}$$

then

$$\kappa h_i(t) = 1 - \left|\frac{t}{t_0}\right|^{2k_i}, \quad h_i(t_0) = 0, \tag{4}$$

where $\kappa = \sqrt{32\pi G_N}$, with G_N—the Newton gravitational constant. The quantized field

$$h_{\mu\nu}(t, \boldsymbol{x}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_1(t) & 0 & 0 \\ 0 & 0 & h_2(t) & 0 \\ 0 & 0 & 0 & h_3(t) \end{pmatrix} \equiv \operatorname{diag}\big(0, h_i(t)\big) \tag{5}$$

is small, as expected, closely to the expansion (reference) point t_0. Using the gauge freedom to satisfy the harmonic gauge condition (see, the second formula in (11)), we gauge transform the gravitational field $h_{\mu\nu}$ as follows,

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{6}$$

where the gauge parameter

$$\xi_\mu(t) = \left(-\frac{1}{2}\int_0^t h(t')\,dt', 0, 0, 0\right), \quad h(t) \equiv h_1(t) + h_2(t) + h_3(t). \tag{7}$$

Then,

$$h'_{\mu\nu}(t, \boldsymbol{x}) = \operatorname{diag}\big(2\dot{\xi}_0(t), h_i(t)\big) = \operatorname{diag}\big(-h(t), h_i(t)\big), \tag{8}$$

and, skipping the prime for simplicity, we have

$$h^\lambda_\lambda(t) = -2h(t), \tag{9}$$

where spacetime indices are being manipulated with the Minkowski metric $\eta_{\mu\nu}$. Now, we should switch from our present $h_{\mu\nu}$ to standard perturbative gravitational variables, i.e. to the "barred" field $\bar{h}_{\mu\nu}$ defined by

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^\lambda_\lambda, \tag{10}$$

and

$$\bar{h}_{\mu\nu}(t, \boldsymbol{x}) = \operatorname{diag}\big(0, h_i(t) - h(t)\big) \quad \text{with } \partial^\mu \bar{h}_{\mu\nu} = 0. \tag{11}$$

The Fourier transform of $\bar{h}_{\mu\nu}$ is

$$\tilde{\bar{h}}_{\mu\nu}(p) \equiv \tilde{\bar{h}}_{\mu\nu}(\omega, \boldsymbol{p}) = (2\pi)^3 \delta^3(\boldsymbol{p}) \operatorname{diag}\big(0, \tilde{h}_i(\omega) - \tilde{h}(\omega)\big), \tag{12}$$

where, for the h_i of the explicit form (4), we have (from now on, we denote classical gravitational fields with the superscript "c")

$$\kappa \tilde{h}^{\mathrm{c}}_i(\omega) = 2\pi\delta(\omega) + 2t_0^{-2k_i} \sin(\pi k_i)\Gamma(2k_i + 1)|\omega|^{-2k_i - 1}. \tag{13}$$

According to (A.8) a one-loop quantum contribution corresponding to the classical metric (12) equals

$$\begin{aligned}\tilde{h}^{\mathrm{q}}_{\mu\nu}(p) &= \frac{\pi\kappa^2 p^2}{2} \log\left(\frac{-p^2}{\Lambda^2}\right)\delta^3(\boldsymbol{p}) \\ &\quad \times \big[(2\alpha_1 \mathbb{E} + 4\alpha_2 \mathbb{P}) \operatorname{diag}\big(0, \tilde{h}^{\mathrm{c}}_i - \tilde{h}^{\mathrm{c}}\big)\big]_{\mu\nu} \\ &= 2\pi\kappa^2\omega^2 \log\left|\frac{\omega}{\Lambda}\right| \delta^3(\boldsymbol{p}) \operatorname{diag}\big(\alpha_2 \tilde{h}^{\mathrm{c}}, \alpha_1 \tilde{h}^{\mathrm{c}}_i - (\alpha_1 + \alpha_2)\tilde{h}^{\mathrm{c}}\big).\end{aligned} \tag{14}$$

Defining the auxiliary function

$$\tilde{h}^{\mathrm{Q}}_i(\omega) \equiv \omega^2 \log\left|\frac{\omega}{\Lambda}\right| \tilde{h}^{\mathrm{c}}_i(\omega), \tag{15}$$

we have

$$\begin{aligned}\kappa \tilde{h}^{\mathrm{Q}}_i(\omega) &= 2\pi\delta(\omega)\omega^2 \log\left|\frac{\omega}{\Lambda}\right| \\ &\quad + 2t_0^{-2k_i} \sin(\pi k_i)\Gamma(2k_i + 1)|\omega|^{-2k_i + 1} \log\left|\frac{\omega}{\Lambda}\right|.\end{aligned} \tag{16}$$

Its Fourier reverse is

$$\begin{aligned}\kappa h^{\mathrm{Q}}_i(t) &= 2t_0^{-2k_i} k_i(2k_i - 1)|t|^{2k_i - 2} \\ &\quad \times \left[\psi(2 - 2k_i) + \frac{\pi}{2}\tan(\pi k_i) - \log|\Lambda t|\right],\end{aligned} \tag{17}$$

where ψ is the digamma function, and according to (14)

$$h^{\mathrm{q}}_{\mu\nu}(t, \boldsymbol{x}) = \left(\frac{\kappa}{2\pi}\right)^2 \operatorname{diag}\big(\alpha_2 h^{\mathrm{Q}}(t), \alpha_1 h^{\mathrm{Q}}_i(t) - (\alpha_1 + \alpha_2) h^{\mathrm{Q}}(t)\big). \tag{18}$$

Performing the gauge transformation in the spirit of (7) we can remove the first (time) component in (18), and (once more, skipping the prime for simplicity) we get a quantum contribution to the Kasner metric

$$h^{\mathrm{q}}_{\mu\nu}(t, \boldsymbol{x}) = \left(\frac{\kappa}{2\pi}\right)^2 \operatorname{diag}\big(0, \alpha_1 h^{\mathrm{Q}}_i(t) - (\alpha_1 + \alpha_2) h^{\mathrm{Q}}(t)\big). \tag{19}$$

Only the "anisotropic" part of (19), i.e.

$$\delta h^{\mathrm{A}}_i = \left(\frac{\kappa}{2\pi}\right)^2 \alpha_1 h^{\mathrm{Q}}_i, \tag{20}$$

can influence the anisotropy of the evolution of the Universe. Since the dependence of δh^{A}_i on k_j is purely "diagonal" (δh^{A}_i depends only on k_j with $j = i$, see (17)), we have the following simple rule governing (de)stabilization of the isotropy: the increasing function $\delta h^{\mathrm{A}}(k)$ implies destabilization (there is a greater contribution of quantum origin to the metric in the direction of a greater classical expansion), whereas the decreasing function implies stabilization. Unfortunately, $\delta h^{\mathrm{A}}(k)$ is not a monotonic function because the digamma function ψ oscillates, and moreover (17) is (in general[1]) a Λ-cutoff dependent function. Nevertheless, if we assume the point of view that it is not necessary to expect or require the stability of the isotropy in the whole domain of the Kasner exponents k_i, but only for some subset of them, considered physically preferred, a definite answer emerges. Since $k = \frac{1}{2}$ corresponds to radiation, and $k = \frac{2}{3}$ corresponds to matter, we could be fully satisfied knowing that $\delta h^{\mathrm{A}}(k)$ is monotonic in the interval $k \in (\frac{1}{4}, 1)$ ($\supset [\frac{1}{2}, \frac{2}{3}]$). Furthermore, since $\alpha_1 > 0$ for any spin (see, Table A.1), $\delta h^{\mathrm{A}}(k)$ is a decreasing function in this interval, implying (quantum) damping of the anisotropy (see, Fig. 1).

[1] The quantum contribution is Λ-cutoff independent for $k_i = \frac{1}{2}$ (a limit in (17) exists), i.e. for pure radiation (see, [5]). Intuitively, it could be explained by the fact that a scale-independent classical source, the photon field, implies vanishing of scale-dependent logarithms (no quantum "anomaly").