Physics Letters B 704 (2011) 260–264

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Thermodynamical analysis on a braneworld scenario with curvature corrections

Ninfa Radicella *, Diego Pavón

Departamento de Física, Universidad Autónoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

ARTICLE INFO

Article history:
Received 26 July 2011
Received in revised form 29 August 2011
Accepted 9 September 2011
Available online 16 September 2011
Editor: T. Yanagida

Keywords:
Cosmology
Thermodynamics
Modified theories of gravity

ABSTRACT

We study the thermodynamics of some cosmological models based on modified gravity, a braneworld with induced gravity and curvature effect. Dark energy component seems necessary if the models are to approach thermal equilibrium in the long run.

1. Introduction

Supernovae type Ia data [1] as well as other observational probes [2] show that the Universe is undergoing an accelerated phase of expansion at present time, a feature that does not emerge from standard cold dark matter model [3]. Attempts to explain such unexpected behaviour go towards modifications of either the geometric part of the Einstein field equations, implying modified theories of gravity, or the matter sector, thus involving new and sometimes weird forms of energy [4]. Such a dark sector seems unavoidable in order to fit present cosmological data.

On the other hand, from the theoretical side, the strong mathematical resemblance between the dynamics of spacetime horizons and thermodynamics is strongly attested [5,6] so that gravitational fields equations can be given a physical interpretation which is thermodynamical in origin. In particular, the Friedmann equations follow from applying the first law to the apparent horizon of an isotropic and homogeneous universe, not only in Einstein gravity, but also in more general Lovelock gravity [7]. Likewise, it seems that a gravitational theory built on the principle of equivalence must be thought of as a macroscopic limit of some underlying microscopic theory, the microscopic structure of spacetime manifesting itself only at Planck scale or near singularities. Also the horizons link some aspects of microscopic physics with the bulk dynamics [8]. It is well known since long that one can define entropy and temperature for a spacetime horizon [9–11]; in fact, many attempts have been done to better understand this link. An instructive example is the case of spherically symmetric horizons in four dimensions, for which Einstein's equations can be interpreted as a thermodynamic relation arising out of virtual displacement of the horizon [12]. Moreover, the same interpretation holds for the case of the Lanczos–Lovelock gravitational theory in D dimensions [13] and explicit demonstration has been given for Friedmann models [14].

In the present Letter, following this deep relation between thermodynamic and gravity, and in particular between entropy and horizons, we argue that some form of dark energy is demanded on thermodynamic grounds.

In order for an isolated system to evolve to thermodynamical equilibrium, the entropy function of the system must show two properties: it must never decrease, i.e. its first derivative with respect to the relevant variable must be non-negative, and convex, i.e., its second derivative must be negative.

This constitutes the hard core of the second law of thermodynamics and it is naturally realised in systems dominated by electromagnetic forces; however it might not be true when gravity plays a role. In fact, the entropy of the system must still increase but it may be grow unbounded: this occurs, in the Newtonian framework, for the Antonov's sphere, the final stage of N gravitating point masses enclosed in a perfectly reflecting, rigid, sphere whose radius exceeds some critical value [15,16]. Nevertheless, when we replace Newtonian gravity by general relativity, a black hole is expected to be formed at the center of the sphere that, though it tends to evaporate, it will likely arrive to an equilibrium state characterised by a state of maximum, finite, entropy.

In any case, in a Friedmann–Robertson–Walker (FRW) cosmology, the Universe seems to behave as an ordinary system whose

* Corresponding author.
E-mail addresses: ninfa.radicella@uab.cat (N. Radicella), diego.pavon@uab.es (D. Pavón).

0370-2693/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2011.09.031

entropy increases towards a maximum value. The latter follows from the observational data on the evolution of the Hubble factor of the FRW metric [17] and from the evolution of the entropy of the apparent horizon, that seems to be the appropriate thermodynamic boundary [18].

The present Letter is a second step of the analysis outlined above. In fact, in a previous paper [19] we showed that an Einstein Universe, as a thermodynamical system, cannot tend to equilibrium in the last stage of expansion unless it accelerates. We have found that this holds true for some modified models that are dynamically equivalent at the background level, nevertheless this does not mean that every accelerating universe is thermodynamically motivated [19].

In this work we study the thermodynamical behaviour of a braneworld model with two correction terms: a four-dimensional curvature on the brane and a Gauss–Bonnet (GB) term in the bulk [20]. The induced gravity (IG) correction arises because the localised matter fields on the brane, which couple to bulk gravitons, can generate via quantum loops a localised four-dimensional world-volume kinetic term for gravitons [21]. On the other hand, a Gauss–Bonnet term naturally appears in an effective action approach to string theory, corresponding to the leading order quantum corrections to gravity [22]. As a result, we have the most general action with second-order field equations in five dimensions [23].

Section 2 introduces the braneworld cosmology with induced gravity and curvature effects. Sections 2.2 and 2.3 focus on the entropy of the horizon and matter components, respectively. The energy components of the Universe are assumed to enter the field equations in the form of perfect fluids, the standard equation of state being true for each of them: $p_i = w_i \rho_i$. Then, in Section 3, the matter component is assumed as cold matter and a Chaplygin gas. The choice of a Chaplygin gas is based on the recent observational result that the equation of state parameter of dark energy can be less than -1 and even display a transient behaviour [2]. This can be achieved either by means of phantom fields, that on the other hand suffer from instabilities [24], or by other approaches that mimic this phantom-like behaviour. In the model under analysis, in which UV modifications are included by considering the stringy effect via the GB term in the bulk, and IR modifications are due to the IG effect, a Chaplygin gas fluid on the brane provides a smooth crossing of the cosmological constant line [25]. In fact, this component is characterised by a cross-over length scale below which the gas behaves as pressureless fluid and above which it mimics a cosmological constant.

Our conclusions are given in Section 4: we find that even this modified theory of gravity needs a component with typical dark energy behaviour in order to satisfy the generalised second law (GSL) and approach thermodynamical equilibrium in the long run.

2. Gauss–Bonnet and induced gravity corrections

The total action of the braneworld model under consideration reads [20]

$$I = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-{}^{(5)}g}\left({}^{(5)}R - 2\Lambda_5 + \alpha\mathcal{L}_{GB}\right) + \frac{1}{2\kappa_4^2}\int d^4x\sqrt{{}^{(4)}g}(R - 2\Lambda_4) + \int d^4x\,\mathcal{L}_m, \tag{1}$$

where $\Lambda_5 < 0$ is the cosmological constant on the bulk and

$$\mathcal{L}_{GB} = {}^{(5)}R^2 - 4{}^{(5)}R^{AB\,(5)}R_{AB} + {}^{(5)}R^{ABCD(5)}R_{ABCD} \tag{2}$$

is the GB correction term, whose coupling constant $\alpha = 1/8g_s^2$ is related to the string energy scale, g_s [20]. The gravitational coupling constants $\kappa_4^2 = 8\pi G_4$ and $\kappa_5^2 = 8\pi G_5$ on the brane and in the bulk, respectively introduce a length scale, the induced gravity cross-over scale, $r = \kappa_5^2/2\kappa_4^2$ and help defining the brane tension, $\lambda = \Lambda_4/\kappa_4^2$. Last term represents matter action.

2.1. Cosmological equations

The metric of homogeneous and isotropic FRW Universe on the brane, with spatial curvature index k, is

$$ds^2 = h_{\mu\nu}\,dx^\mu\,dx^\nu + \tilde{r}^2\left[d\theta^2 + \sin\theta^2\,d\phi^2\right], \tag{3}$$

where $\tilde{r} = a(t)r$, the two-dimensional metric is $h_{\mu\nu} = \mathrm{diag}(-1, a^2/(1-kr^2))$ with $x^0 = t$ and $x^1 = r$. This allows the explicit evaluation of the radius of the apparent horizon (a marginally trapped surface with vanishing expansion) determined by the relation $h^{\mu\nu}\partial_\mu\tilde{r}\partial_\nu\tilde{r} = 0$ [26] that gives

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}. \tag{4}$$

Friedmann's equation on the brane is

$$-\frac{1}{r}\left[1 + \frac{8}{3}\alpha\left(H^2 + \frac{k}{a^2} + \frac{\Phi_0}{2}\right)\right]\left(H^2 + \frac{k}{a^2} - \frac{\Phi_0}{2}\right)^{1/2} = -\frac{k_4^2}{3}\left(\sum_i \rho_i + \lambda\right) + H^2 + \frac{k}{a^2}, \tag{5}$$

in which, assuming there is no black hole in the bulk, $\Phi_0 = \frac{1}{4\alpha}(-1 + \sqrt{1 + \frac{4}{3}\alpha\Lambda_5})$ and matter field are supposed to be perfect fluids with energy density ρ_i so that in order to describe completely the cosmological dynamics on the brane we can use the energy conservation law

$$\dot{\rho}_i + 3H\rho_i(1 + w_i) = 0, \tag{6}$$

where $w_i = p_i/\rho_i = \mathrm{const.}$

2.2. Entropy of the apparent horizon

The entropy on the apparent horizon is given by [27–29]:

$$S_A = 4\pi\left[\frac{1}{2G_4}\int_0^{\tilde{r}_A}\tilde{r}_A\,d\tilde{r}_A + \frac{1}{2G_5}\int_0^{\tilde{r}_A}\frac{\tilde{r}_A^2\,d\tilde{r}_A}{\sqrt{1 - \Phi_0\tilde{r}_A^2}} + \frac{2\alpha}{G_5}\int_0^{\tilde{r}_A}\frac{2 - \Phi_0\tilde{r}_A^2}{\sqrt{1 - \Phi_0\tilde{r}_A^2}}\,d\tilde{r}_A\right]. \tag{7}$$

Its derivatives with respect to the scale factor a, that will be denoted by a prime, can be calculated and simplified by using Eqs. (5)–(6)

$$S'_A = 8\pi^2\tilde{r}_A^4\frac{\sum_i(\rho_i + p_i)}{a} = 8\pi^2\tilde{r}_A^4\frac{\rho_T(1 + w_T)}{a}, \tag{8}$$

$$S''_A = \frac{S'}{a}\left[4\frac{\tilde{r}'_A a}{r_A} - (3w_T + 4)\right], \tag{9}$$

where $w_T = p_T/\rho_T = \sum_i p_i/\sum_i \rho_i$.

It can be easily checked that the entropy grows provided the fluids component of the Universe satisfy $w_T > -1$. In order to evaluate the second derivative we make use of the late time behaviour of this brane cosmology with curvature correction that, as