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Gauge invariance in braneworld scenarios

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ABSTRACT

Rubakov and Shaposhnikov (RSH), in a seminal paper, discussed the possibility that particles are confined in a potential well. This is considered as the first mention to the today's idea that we live in a brane, i.e., the braneworld concept. In this work we show precisely that the proposed RSH model has a gauge invariant equivalent action and we discuss it in the light of braneworld structure. We analyzed the intrinsic features of both models trying to disclose new properties within RSH braneworld theory.

1. Introduction

Years ago [1] Rubakov and Shaposhnikov (RSH)[1] proposed a high-dimensional model in which ordinary matter is confined in a $(3+1)$-dimensional subspace, or a 3-brane. This extra-dimensional subspace is not necessarily compact [2] (see also [3,4]), contradicting the work of Kaluza–Klein [5].

The RSH model is considered in the literature the first-step given to introduce the concept of braneworld, in modern nomenclature, or domain wall at that time. Namely, in a braneworld, particles are trapped in a four-dimensional Minkowski subspace under the influence of a potential V.

The Randall–Sundrum (RS) models in five dimensions can be fathomed as the simplest braneworld framework with an extra spatial dimension. The massless graviton mode reproduces the standard Newtonian gravity on the 3-brane. The Kaluza–Klein modes give corrections to Newton's force law [6]. In RS model an effective dimensional reduction happens without the need of compactifying the fifth dimensions [7]. There are generalizations of the RS model in higher spacetime dimension with applications in gravitational physics [8] and cosmology [9].

The braneworld scenario has been an interesting field to understand some of the most relevant questions concerning theoretical physics. The universe evolution has been a main question and some theories have been proposed in this sense. Some concise proposals about this research area generally guide us by a multidimensional scenario. The idea that such extra dimensions may not be compact or large is allowing new considerations about the hierarchy problem and consequently about the cosmological constant [1,12–23].

In an extra-dimensional scenario [24] the Universe can be considered as a four-dimensional membrane embedded in a higher dimensional spacetime. The membrane encompasses the standard model of particles and the gravitation is free to propagate through the extra-dimensional in the whole spacetime (bulk).

We organized this work in order to provide the reader, in Section 2, with a brief description of the Noether embedding method to construct gauge invariant equivalent models. In Section 3, the reader will find a brief review of the RSH model and its main features. In Section 4 we will demonstrate that the RSH model has a gauge invariant formulation. We will also make a comparative analysis between both models. The conclusions can be found in Section 5.

2. Noether embedding method

In a seminal paper [25], Deser and Jackiw used the master action concept to show the dynamical equivalence between the self-dual (SD) and Maxwell–Chern–Simons (MCS) theories. The authors demonstrated precisely the existence of a hidden symmetry in the self-dual model. After this result, the master action procedure was used to disclose the physics behind planar physics phenomena and bosonization, only to mention a few. Concerning the last one, it is worth to explain that it is a procedure that expresses a theory of interacting fermions in terms of free bosons. In $D=2$ this

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[1] We will use this form to designate these authors since the one used to indicate the Randall–Sundrum work is largely represented in the literature by RS.

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approach reveals the dual equivalence feature of these representations [26–30] being extended to higher dimensions [31,32].

Motivated by this equivalence between SD and MCS models it was natural to ask if there is another way to obtain analogous and new results. In papers analyzing the existence of hidden gauge symmetries inside second-class systems, it was proved that non-invariant models (second-class) are in fact equivalent to gauge invariant systems (first-class). These happens under certain gauge fixing conditions. The main advantage of having a gauge theory is the fact that we can establish chains of equivalence between different models through different gauge fixing conditions.

The Noether embedding technique [33] is based on the traditional idea of a local lifting of a global symmetry and may be realized by an iterative embedding of Noether counter terms. This technique was originally explored in the soldering formalism context [34–36] and was explored in [37–39] since it seems to be the most appropriate technique for non-Abelian generalization of the dual mapping concept.

The method is an iterative procedure and therefore we will describe it as an algorithm as if we were describing a computer program. The first step entails the imposition of a trivial local gauge transformation concerning the zeroth-Lagrangian, which is how we will call the original and so far untouched Lagrangian. Of course this zeroth-Lagrangian is not gauge invariant under this imposed gauge transformation. The computation of its variation permit us to construct the Noether currents. So, the variation of the zeroth-Lagrangian can be written as a sum of the Noether currents coupled with the local gauge parameter. This basically closes the first iteration.

The second one begins with the introduction of auxiliary fields interacting with the Noether currents computed during the first iteration. Hence, the now first-Lagrangian can be written as a sum of the zeroth-Lagrangian plus the terms with the coupling of Noether currents with the auxiliary fields. We perform the variation of this first-Lagrangian to verify if it is gauge invariant. If it is not gauge invariant yet, we have to introduce auxiliary fields (not necessarily new ones) again in order to try to obtain gauge invariance. If we did not obtain that, a new Lagrangian has to be constructed and the process continues until we have gauge invariance.

Of course, some missing details or comprehension will be found when we apply the method in RSH model, in a few moments. The interested reader that is not pleased with this brief explanation can find more details and applications in [33,37–39].

We have to add an important comment here. The Noether embedding algorithm can be applied in Abelian and non-Abelian theories independent of their dimensions. Hence, it is a perfect procedure to work with field theory with extra dimensions, which is our case here.

3. The RSH model

To make a brief review of the RSH model, let us write the quantum field model [1] with Lagrangian as originally written by the authors, namely,

$$\mathcal{L} = \frac{1}{2}\partial_A\phi\partial^A\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4, \quad A = 0,\dots,4 \tag{1}$$

which describes one scalar field ϕ in $(4+1)$-dimensional spacetime $M^{(4,1)}$ with metric given by $g_{AB} = \mathrm{diag}(1,-1,-1,-1,-1)$. The classical equation of motion have a domain wall solution $\phi^{cl}(x^4)$, which is a $(1+1)$-dimensional kink, independent of three spatial coordinates

$$\phi^{cl}(x^4) = \frac{m}{\sqrt{\lambda}}\tanh\left(\frac{mx^4}{\sqrt{2}}\right). \tag{2}$$

This solution furnish a potential well. It is narrow in the fourth direction if m is sufficiently large [1].

In [1] it was investigated the possibility of trapping particles with spin 0 and particles with spin 1/2. Here we will study the spin 0 case. For particles with spin 0 [10] the equation of motion for the field $\phi' = \phi - \phi^{cl}$ is

$$\partial_A\partial^A\phi' + m^2\phi' + 3\lambda\big(\phi^{cl}\big)^2\phi = 0,$$

and it can be shown that there are three types of perturbations [1,10,11].

The first one is confining, i.e., the particles are trapped inside the wall

$$\phi'(x^A) = \left(\frac{d\phi^{cl}}{dx^4}\right)\exp\big[i\big(-\vec{k}\cdot\vec{x} + Ex^0\big)\big], \tag{3}$$

with the energy $E^2 = \vec{k}^2$. The second one, where the perturbations are also confined, is

$$\phi' = u\big(x^4\big)\exp\big[i\big(-\vec{k}\cdot\vec{x} + Ex^0\big)\big], \tag{4}$$

where $u(x^4)$ is a normalizable solution of

$$-\partial_4^2 u - m^2 u + 3\lambda\big(\phi^{cl}\big)^2 u = \frac{3}{2}m^2 u$$

with energy $E = m^2 + \frac{3}{2}m^2$. And the third one where there exist perturbations which are not confined.

We will show that the spectrum of the perturbations of the equivalent action to (1) in the presence of a brane is given by the second type of perturbation. Of course, as we are considering confined particles, only the first two types are relevant.

4. The gauge invariant RSH model

Let us now use the Noether embedding formalism, described in Section 2, to investigate the gauge invariance of the RSH model. As explained in Section 2, the first step of the algorithm is to impose the most trivial local gauge transformation

$$\delta\phi = \alpha\big(x^A\big) \tag{5}$$

where α is the local gauge parameter depending on x^A. Our final objective is to obtain a final gauge invariant action and to analyze its implications in a braneworld configuration.

The RSH model is given by the action (1) and its variation is

$$\delta\mathcal{L}_0 = J_1\partial^A\alpha + J_2\alpha \tag{6}$$

where

$$J_1 = \partial_A\phi \quad \text{and} \quad J_2 = -m^2\phi - \lambda\phi^3 \tag{7}$$

are the Noether currents. As can be seen obviously from (6), $\delta\mathcal{L}_0$ is not zero and we have to perform the next step of the algorithm, which is to introduce auxiliary fields and to construct the first-Lagrangian. Notice that we will depict from now on some details concerning the method that where not shown in Section 2 in the name of a simple explanation of the algorithm. But these hidden details are very simple and do not jeopardize the explanation given in Section 2.

Hence, we can write that

$$\mathcal{L}_1 = \mathcal{L}_0 - J_1 D_1 - J_2 D_2, \tag{8}$$

where D_1 and D_2 are the auxiliary fields that will be eliminated from the final Lagrangian through their equations of motion. Let us impose also that $\delta D_1 = \partial^A\alpha$ and $\delta D_2 = \alpha$ so that